Effective Field Theory for Compact Binary

致密双星有效场论

Dynamics

动力学

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Abstract

摘要

I review the effective field theory (EFT) description of gravitating compact objects. The focus is on kinematic regimes where gravity is perturbative, in particular the adiabatic inspiral phase relevant to gravitational wave detection. For such configurations, there is a hierarchy of length scales which all play a role in the dynamics, ranging from the gravitational radius to the size of the objects, their typical orbital separation, and finally the wavelength of the radiation emitted by the system. To disentangle these scales, and to achieve manifest power counting in the expansion parameter, it is necessary to construct a tower of EFTs of gravity, each coupled to distinct line defect localized degrees of freedom. I describe the relevant effective theories at each scale as well as the matching between these theories across each physical threshold. While the main applications of these methods are to classical dynamics, quantum gravity effects, e.g., Hawking graviton exchange, can be systematically incorporated if the momentum transfers are small compared to the Planck mass.

我将综述引力致密天体的有效场论 (EFT) 描述。本文重点关注引力满足微扰条件的运动学区域，尤其是和引力波探测相关的绝热旋进阶段。对于这类构型，动力学过程涉及多个长度尺度的层级结构，范围从引力半径、天体自身尺寸、天体典型轨道间距，一直到该系统辐射的引力波波长。为了理清这些尺度，并在展开参数中实现显式幂计数，需要构建一套引力有效场论体系，每个有效场论耦合到不同的线缺陷局域自由度。我将介绍各个尺度对应的相关有效理论，以及不同理论在各个物理阈值间的匹配。这些方法主要应用于经典动力学，但如果动量转移远小于普朗克质量，量子引力效应例如霍金引力子交换也可以被系统地纳入考量。

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关键词

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引力有效场论-黑洞-引力波辐射-后牛顿展开-后闵可夫斯基展开-密近双星系统

Introduction

引言

The observation by LIGO [1] in 2015 of gravitational waves sourced by the merger of two extra-galactic black holes [2] has brought renewed focus on understanding the gravitational dynamics of bound compact (black hole or neutron star) binary systems. When the objects are separated by distances close to the typical Schwarzschild radius $r_s = 2G_N M$, they are in the regime of large spacetime curvature and strong gravitational fields, quantitatively tractable only by the methodology of numerical general relativity [80]. However, in the early adiabatic inspiral stage, when the orbital separation is large, $r \gg r_s$, the evolution of the system is slow and admits a systematic expansion in powers of $r_s/r \ll 1$. By the virial theorem of Newtonian gravity, the adiabatic inspiral is necessarily characterized by non-relativistic orbits, with typical relative velocities (Conventions: We adopt units where $c = \hbar = 1$ and define $\eta_{\mu\nu} = \mathrm{diag}\,(1, -1, -, 1-1)$, $R^{\mu}{}_{\nu\rho\sigma} = \partial_\rho \Gamma^{\mu}{}_{\sigma\nu} + \Gamma^{\mu}{}_{\rho\lambda}\Gamma^{\lambda}_{\sigma\nu} - (\rho \leftrightarrow \sigma)$.) of order

2015 年 LIGO[1] 观测到来自两个河外黑洞并合产生的引力波 [2]，这令人们重新聚焦于研究束缚致密双星 (黑洞或中子星) 系统的引力动力学。当两天体间距接近典型史瓦西半径 $r_s = 2G_N M$ 时，系统处于大时空曲率与强引力场区域，只有数值广义相对论方法 [80] 才能定量求解。但在早期绝热旋近阶段，轨道间距较大，$r \gg r_s$ ，系统演化缓慢，可以按 $r_s/r \ll 1$ 的幂次做系统展开。根据牛顿引力位力定理，绝热旋近必然满足非相对论轨道，典型相对速度 (约定: 我们采用 $c = \hbar = 1$ 的单位制，并定义 $\eta_{\mu\nu} = \mathrm{diag}\,(1, -1, -, 1-1)$, $R^{\mu}{}_{\nu\rho\sigma} = \partial_\rho \Gamma^{\mu}{}_{\sigma\nu} + \Gamma^{\mu}{}_{\rho\lambda}\Gamma^{\lambda}_{\sigma\nu} - (\rho \leftrightarrow \sigma)$ 。) 量级为

$$v^2 \sim \frac{G_N M}{r} \sim \frac{r_s}{r} \ll 1$$

In order to optimize the detection of merger signals buried in the noisy gravitational wave data, for the purposes of parameter extraction (binary masses, spins, etc.), and to interface with numerical relativity simulations, it is crucial to carry out the analytical expansion of Einstein's equations to rather high order in powers of the velocity v . Accurate theoretical wave templates are needed to at least order v^{10} [119] relative to the zeroth-order solution, consisting of predominantly quadrupolar gravitational radiation sourced by nearly Newtonian orbits. The traditional approach to the ”post-Newtonian” (PN) expansion of the solution to Einstein equations as a perturbative series in powers v has a long history; see [12,103] for reviews and complete list of references.

> 为了优化探测掩埋在引力波噪声数据中的并合信号、提取参数 (双星质量、自旋等)，并对接数值相对论模拟，必须将爱因斯坦方程按速度 v 的幂次解析展开到相当高的阶次。相对于零阶解 (近牛顿轨道产生的主导四极引力辐射)，至少需要精确到 v^{10} 阶的理论波形模板 [119]。爱因斯坦方程解按 v 幂次做微扰级数展开的传统“后牛顿”(PN) 方法由来已久，综述与完整参考文献可见 [12,103]。

A more modern approach to perturbative gravitational dynamics, first proposed in [51], recasts the PN expansion in a language more familiar to particle physicists, as an effective field theory (EFT) of self-interacting gravitons coupled to classical worldline sources. In this formulation, the graviton fluctuations about a fixed yet arbitrary configuration of point-like defects are integrated out, resulting in an effective action functional whose extrema yield the classical dynamics of the binary system. In this way, the theory systematically captures both the effects of conservative gravitational forces (”potentials”) and radiation reaction on the evolution of the orbits. Once the classical extrema have been determined, one uses them to calculate the one-point function of the graviton $\langle h_{\mu\nu}\rangle$ sourced by the binary, which encodes the waveform seen by detectors placed at asymptotic future null infinity $\mathcal{I}^+$.

> 更现代的微扰引力动力学方法最早由文献 [51] 提出，它将后牛顿展开改写为粒子物理学家更熟悉的形式: 即自相互作用引力子耦合经典世界线源的有效场论 (EFT)。在该框架中，我们对任意固定点缺陷构型周围的引力子涨落做积分，得到有效作用量泛函，其极值给出双星系统的经典动力学。通过这种方式，该理论可以系统地同时捕获保守引力 (“势”) 效应与辐射反作用对轨道演化的影响。确定经典极值后，就可以用它计算双星产生的引力子 $\langle h_{\mu\nu}\rangle$ 单点函数，该函数编码了未来类空无穷远 $\mathcal{I}^+$ 处探测器观测到的波形。

The EFT approach to gravitationally bound systems relies on the observation that binary dynamics in the PN regime involves a hierarchy of well-separated length scales

> 引力束缚系统的有效场论方法基于如下观测: 后牛顿区域的双星动力学存在多个分隔良好的长度标度层级

$$r_s = \text{ gravitational radius } \sim 2G_N M$$

$$\mathcal{R} = \text{ typ. size of binary constituents,}$$

$$r = \text{ typ. orbital radius } \sim v^{-2} r_s \text{ ,}$$

$$\lambda = \text{ typ. radiation wavelength } \sim v^{-1} r$$

with

满足

$$r_s \lesssim \mathcal{R} \ll r \ll \lambda$$

First, we assume that at distances larger than the Planck length, $\ell_{\mathrm{Pl}} = m_{\mathrm{Pl}}^{-1} \equiv \sqrt{32\pi G_N} \sim 10^{-19}\mathrm{GeV}^{-1}$, gravity is described by the Einstein-Hilbert Lagrangian [36]

首先，我们假设在距离大于普朗克长度 $\ell_{\mathrm{Pl}} = m_{\mathrm{Pl}}^{-1} \equiv \sqrt{32\pi G_N} \sim 10^{-19}\mathrm{GeV}^{-1}$ 的尺度下，引力由爱因斯坦-希尔伯特拉格朗日量描述 [36]

$$S_{EH} = -2m_{\mathrm{Pl}}^2 \int d^4x\sqrt{g}R + \cdots , \tag{1}$$

expanded around a fixed background, e.g., the Minkowski vacuum, $g_{\mu\nu} = \eta_{\mu\nu} + \ h_{\mu\nu}/m_{\mathrm{Pl}}$. We take the point of view [34] that Eq. (1) represents the unique [112, 113] low energy effective theory of quantum gravity, with well-defined Feynman rules [30-32, 39, 64], with sensible infrared (IR) behavior [114] and ultraviolet (UV) renormalization properties [62,109]. We have suppressed in Eq. (1) an infinite tower of local higher curvature terms, generated by loop corrections, which are assumed to be kinematically suppressed by powers of $(E_{\mathrm{CM}}/m_{\mathrm{Pl}})^2 \ll 1$ in a typical process. More precisely, for the applications of interest in this review, we consider the classical limit with $\ell_{Pl} \to 0$ and gravitational radius $r_s \sim G_N E_{\mathrm{CM}}$ held fixed to be somewhat smaller than the physical radius $\mathcal{R} \gg \ell_{\mathrm{Pl}}$ of the compact objects, in which case the higher-order terms in Eq. (1) give rise to corrections suppressed by powers of $(\ell_{\mathrm{Pl}}/r_s)^2 \ll 1$.

围绕固定背景展开，例如闵可夫斯基真空，$g_{\mu\nu} = \eta_{\mu\nu} + \ h_{\mu\nu}/m_{\mathrm{Pl}}$。我们认同文献 [34] 的观点: 式 (1) 是量子引力唯一的低能有效理论 [112, 113]，具有明确定义的费曼规则 [30-32, 39, 64]，合理的红外 (IR) 行为 [114] 和紫外 (UV) 重整化性质 [62,109]。我们在式 (1) 中省略了由圈修正产生的无穷多局域高曲率项，通常认为这些项在一般过程中会被 $(E_{\mathrm{CM}}/m_{\mathrm{Pl}})^2 \ll 1$ 的幂次运动学压低。更准确地说，对于本综述关注的应用场景，我们考虑的经典极限是: $\ell_{Pl} \to 0$ 和引力半径 $r_s \sim G_N E_{\mathrm{CM}}$ 保持为小于致密天体的物理半径 $\mathcal{R} \gg \ell_{\mathrm{Pl}}$ ，此时式 (1) 中的高阶项带来的修正会被 $(\ell_{\mathrm{Pl}}/r_s)^2 \ll 1$ 的幂次压低。

For a compact object of a given mass, the scale $\mathcal{R}$ depends on the detailed internal structure via a thermodynamic equation of state. By definition, a compact object is one with $\kappa = \mathcal{R}/2G_N M \gtrsim \mathcal{O}(1)$ (e.g., $\kappa_{BH} = 1$ for a Schwarzschild black hole, while $\kappa_{NS} \sim \mathcal{O}(10)$ for neutron stars). Even though the typical energy scale E_{CM} is super-Planckian, the orbital separation $\sim r$ in a binary encounter is taken to be large, $r \gg \mathcal{R} \gg r_s$, so that observables at scales r can be computed by treating the compact constituents as point defects whose worldlines deflect via graviton exchange and which source and absorb radiation. This theory of classical worldlines coupled to gravitons is suitable for computing gravitational radiation in compact binaries, as an expansion in powers of r_s/r (formally, this is an expansion in powers of G_N , the so-called ”post-Minkowskian” (PM) expansion of general relativity). In such a kinematic regime, the relevant graviton modes have typical four-momenta $k^\mu \sim 1/r$.

对于给定质量的致密天体，尺度 $\mathcal{R}$ 通过热力学物态方程依赖于具体内部结构。根据定义，致密天体满足 $\kappa = \mathcal{R}/2G_N M \gtrsim \mathcal{O}(1)$ (例如史瓦西黑洞满足 $\kappa_{BH} = 1$，中子星满足 $\kappa_{NS} \sim \mathcal{O}(10)$)。尽管典型能量尺度 $E_{\rm CM}$ 是超普朗克的，但双星交会中的轨道间距 $\sim r$ 很大，满足 $r \gg \mathcal{R} \gg r_s$，因此我们可以将致密组元处理为点缺陷来计算 r 尺度上的可观测量: 它们的世界线因引力子交换偏转，并且可以辐射和吸收引力辐射。这种经典世界线与引力子耦合的理论适合计算致密双星的引力辐射，可以按 r_s/r 的幂次展开 (形式上这是按 G_N 的幂次展开，即广义相对论中所谓的“后闵可夫斯基”(PM) 展开)。在此运动学区域，相关引力子模式的典型四动量满足 $k^\mu \sim 1/r$.

For applications to gravitational wave astronomy, one is in addition interested in adiabatic binary inspirals, with bound non-relativistic orbits, $v \ll 1$. Consequently, there is now an additional separation of scales between orbital dynamics at the scale r and radiation at a characteristic wavelength set by the multipole expansion, $\lambda \sim \ r/v \gg r$. Thus, the various scales in the problem become correlated, in the sense that the single expansion parameter v controls the relative contribution of physics arising at widely separated scales, $r_s \sim v^2 r \sim v^3 \lambda$.

对于引力波天文学的应用,人们还额外关注绝热双星并合过程,该过程是束缚的非相对论轨道,$v \ll 1$。因此，尺度 r 的轨道动力学与多极展开设定特征波长的辐射 $\lambda \sim \ r/v \gg r$ 之间存在额外的标度分离。因此，问题中的各类尺度相互关联，即单个展开参数 v 控制着来自差异很大的尺度的物理过程的相对贡献，$r_s \sim v^2 r \sim v^3 \lambda$。

In order to disentangle the various effects at a given order in v , it is natural to organize the physics in terms of a tower of Wilsonian EFTs of gravity [51, 53] , as depicted in Fig. 1. Reformulating binary dynamics within the framework of EFT then leads to conceptual and technical simplifications, for exactly the same reasons as in the well-established applications of EFT to systems that do not contain gravity (e.g., in high energy physics or condensed matter):

为了理清 v 给定阶下的各类效应，很自然地将物理按引力的威尔逊有效场论 (EFT) 序列 [51, 53] 组织，如图 1 所示。在 EFT 框架内重新表述双星动力学，会带来概念和技术上的简化，原因和不含引力的系统 (例如高能物理或凝聚态系统) 中成熟的 EFT 应用完全一致:

- Power counting: The Wilson coefficients of local operators in the effective Lagrangian depend only on the UV energy scale Λ_{UV} corresponding to modes that propagate over short distances which have been integrated out from the Lagrangian. Power counting of corrections to observables defined at an IR energy scale Λ_{IR} , in the expansion parameter $\Lambda_{IR}/\Lambda_{UV} \ll 1$, is manifest.

  - 幂次计数: 有效拉格朗日量中局部算符的威尔逊系数仅依赖于对应短距离传播模式的紫外能标 Λ_{UV}，这些模式已从拉格朗日量中积分积出。对定义在红外能标 Λ_{IR} 的可观测量的修正，其幂次计数在展开参数 $\Lambda_{IR}/\Lambda_{UV} \ll 1$ 下是明确显现的。

- Analyticity of short-distance contributions: UV effects are in one-to-one correspondence with local operators in the effective Lagrangian. Thus at any given order in $\Lambda_{IR}/\Lambda_{UV}$, the most general Lagrangian that is consistent with the symmetries of the relevant degrees of freedom accessible to experiments at energies $\sim \Lambda_{IR}$ necessarily describes the UV physics in a model-independent way. For suitably defined observables, these short-distance contributions depend analytically on the kinematics.

- 短程贡献解析性: 紫外效应与有效拉格朗日量中的局部算符一一对应。因此在 $\Lambda_{IR}/\Lambda_{UV}$ 的任意给定阶，满足能量 $\sim \Lambda_{IR}$ 实验可及相关自由度对称性的最一般拉格朗日量，必然以模型无关的方式描述紫外物理。对于适当定义的可观测量，这些短程贡献对运动学是解析依赖的。

- Renormalization group (RG) evolution: Non-analytic contributions, in the form of (potentially) large logarithms $\ln \Lambda_{UV}/\Lambda_{IR} \gg 1$, can be understood as the RG evolution of the EFT Wilson coefficients from a matching scale $\mu \sim \Lambda_{UV}$ where the EFT is defined (by matching to a more complete microscopic theory) down to the IR at a scale $\mu \sim \Lambda_{IR}$. The scaling dimensions of the Wilson coefficients are calculable in the EFT, and non-analytic effects in $\Lambda_{IR}/\Lambda_{UV}$ are universal, independent of the detailed microscopic physics that might not yet be experimentally resolvable.

- 重整化群 (RG) 演化: 非解析贡献以 (潜在的) 大对数 $\ln \Lambda_{UV}/\Lambda_{IR} \gg 1$ 的形式存在，可以理解为有效场论 (EFT) 威尔逊系数从 EFT 定义的匹配能标 $\mu \sim \Lambda_{UV}$ (通过匹配到更完整的微观理论) 到红外能标 $\mu \sim \Lambda_{IR}$ 的 RG 演化。威尔逊系数的标度维数可在 EFT 中计算，且 $\Lambda_{IR}/\Lambda_{UV}$ 中的非解析效应是普适的，不依赖尚未被实验分辨的微观细节物理。

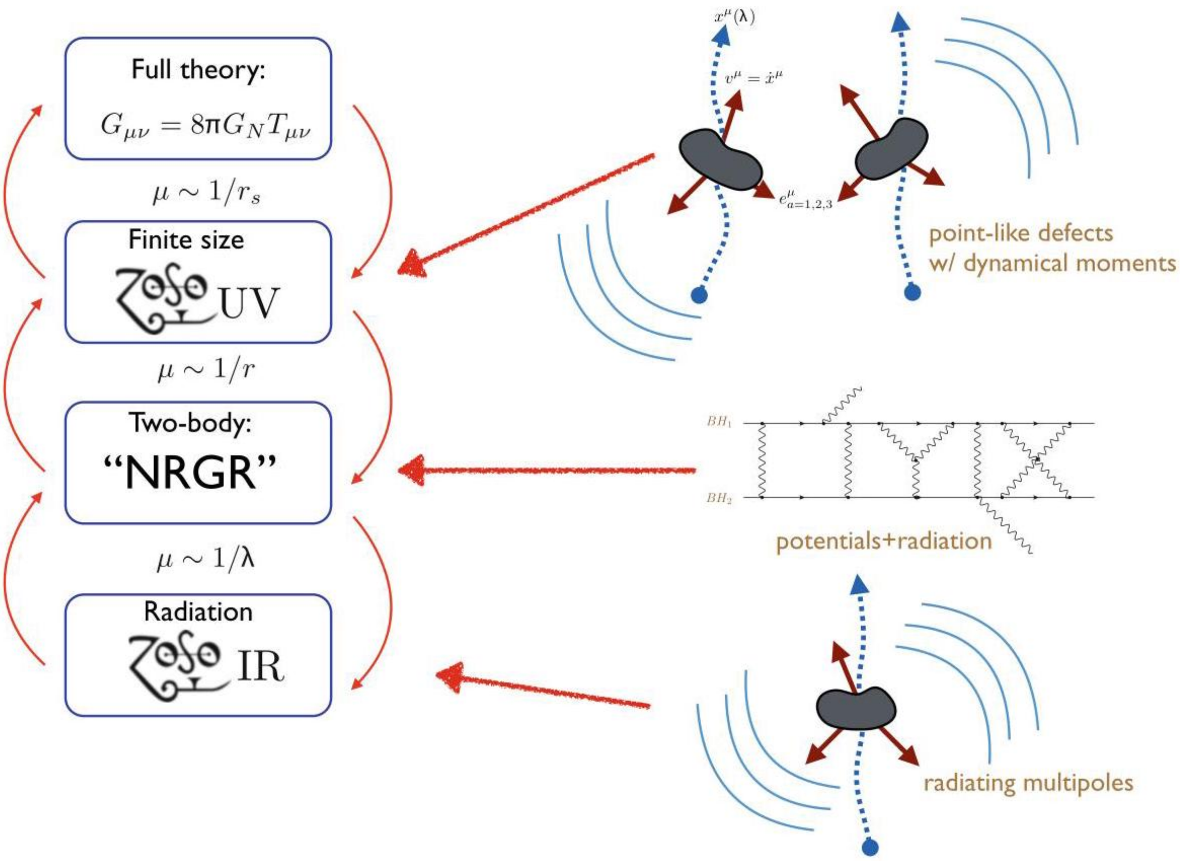


Fig. 1 Tower of gravity EFTs for non-relativistic compact bound states

图 1 非相对论紧致束缚态的引力有效场论层级

The goal of this chapter is to present a detailed overview of the EFT interpretation of binary dynamics, in the regime $r_s/r \ll 1$ where perturbation theory applies. In section "The One-Body Sector" we integrate out the internal structure of an isolated compact object. We show how, at distance scales larger than the radius $\mathcal{R}$, the response to external gravitational fields is systematically encoded in the Wilson coefficients of a local worldline action, suppressed by powers of $\mathcal{R}$. For simplicity we assume in Sec. that the internal dynamics is gapped, i.e., that there is no absorption or emission of bulk gravitons.

本章的目标是详细介绍微扰论适用区域 $r_s/r \ll 1$ 中双星动力学的 EFT 诠释。在“单体区域”一节，我们积出孤立紧致天体的内部结构。我们展示了，在半径 $\mathcal{R}$ 以上的距离尺度，对外部引力场的响应被系统编码在局部世界线作用量的威尔逊系数中，被 $\mathcal{R}$ 的幂次压低。为简化起见，我们在该节假设内部动力学有能隙，即不存在 bulk 引力子的吸收或辐射。

In section ”Perturbative Binary Dynamics” we formulate the two-body problem as a theory of gravitons coupled to the compact object worldlines. Because of the hierarchy between conservative orbital dynamics at scales $\sim r$ and radiative effects at wavelengths $\lambda \sim r/v \gg r$ in the non-relativistic limit, ensuring manifest scaling in powers of velocity requires the construction of two a priori independent EFTs of gravity, whose structure is discussed in sections ”NRGR” and ”Radiative Corrections and TSE IR, ”respectively. One EFT, NRGR (section ”NRGR”), is a theory of potential and radiation graviton modes which is operative at distance scales between $\mathcal{R}$ and r, while at distances $\gtrsim r$, the system is described by an EFT of nonlinear radiation coupled to a set of multipole moments localized on a ”defect” worldline (section ”Radiative Corrections and 26 IR,”).

在“微扰双星动力学”一节，我们将二体问题表述为引力子耦合到紧致天体世界线的理论。非相对论极限下，守恒轨道动力学的尺度 $\sim r$ 与辐射效应的波长 $\lambda \sim r/v \gg r$ 之间存在层级，为了保证速度幂次的显式标度，需要预先构造两个独立的引力 EFT，它们的结构分别在“NRGR”和“辐射修正与 TSE 红外”两节讨论。其中一个 EFT 即 NRGR(“NRGR”节) 是势引力子模式与辐射引力子模式的理论，适用于 $\mathcal{R}$ 到 r 之间的距离尺度；而在距离 $\gtrsim r$ 处，系统由非线性辐射耦合到局域在“缺陷”世界线上的多极矩的 EFT 描述 (“辐射修正与红外 26”节)。

Section ”Time Non-locality: Radiation Reaction, Black Hole Event Horizons” provides a survey of various types of nonlocal in time phenomena associated with binary dynamics. First, in section ”Radiation Reaction in ZSO IR” we explain how the EFT consistently predicts the backreaction of the emitted gravitational waves (radiation reaction) on the evolution of the non-relativistic orbits. In section ”Event Horizon Dynamics in [239] UV,” we account for the non-trivial effects associated with the event horizon of a black hole in a binary bound state. Because the excitations associated with perturbations of the black hole horizon propagate over distance scales of order in the Schwarzschild radius r_s, finite size effects are no longer gapped, and the local worldline description presented in section ”The One-Body Sector” does not correctly capture the IR physics. Nevertheless, we show in section ”Event Horizon Dynamics in ZSOUV”, that finite size dissipative effects on the binary system, such as the absorption of energy and momentum by the horizon, and even super-radiant amplification of radiation, can still be described model-independently within an EFT that contains additional worldline localized degrees of freedom whose coupling to gravitons is constrained by diffeomorphism invariance. In this section, we also provide a gauge invariant definition of the ”Love numbers” that characterize the tidal response of a black hole, using the language of (linear) response theory.

章节“时间非定域性: 辐射反作用、黑洞事件视界”概述了与双星动力学相关的各类时间非定域现象。首先，在“ZSO 红外区的辐射反作用”小节中，我们阐释了有效场论 (EFT) 如何一致地预言发射引力波的反作用 (即辐射反作用) 对非相对论轨道演化的影响。在“239 紫外区的事件视界动力学”小节中，我们讨论了束缚双星系统中与黑洞事件视界相关的非平凡效应。由于黑洞视界扰动对应的激发过程在施瓦西半径 r_s 量级的距离尺度上传播，有限尺寸效应不再存在能隙，因此“单体 sector”章节给出的局域世界线描述无法正确描述红外物理。尽管如此，我们在“ZSOUV 区的事件视界动力学”小节中表明，双星系统的有限尺寸耗散效应——例如视界对能量动量的吸收，甚至超辐射放大辐射——仍可以在有效场论框架下进行不依赖模型的描述: 该 EFT 引入了额外的局域在世界线上的自由度，这些自由度与引力子的耦合受微分同胚不变性约束。在本小节中，我们还利用 (线性) 响应理论的语言，给出了表征黑洞潮汐响应的“洛夫数”的规范不变定义。

While not relevant to phenomenology, the EFTs presented in this review are also capable of capturing quantum corrections to gravitationally bound states of black holes. To illustrate this point, in section "Quantum Effects" we extend the formalism of section "Event Horizon Dynamics in TSUV" to incorporate the effects of Hawking radiation [67] on black hole two-body interactions. As a simple example, we analyze how the exchange of virtual Hawking radiation leads to new calculable features in the inelastic scattering of an elementary particle of mass $\ll m_{\rm Pl}$ by a Schwarzschild black hole, giving rise to effects at the same order in $(E_{\rm CM}/m_{\rm Pl})^2$ as the leading quantum corrections to scattering in quantum gravity due to graviton loops, of the sort first studied in [11,33,34].

虽然不影响唯象学，但本综述介绍的有效场论同样可以描述黑洞引力束缚态的量子修正。为说明这一点，我们在“量子效应”小节中将“TSUV 区的事件视界动力学”的形式体系进行扩展，纳入了霍金辐射 [67] 对黑洞两体相互作用的影响。作为一个简单例子，我们分析了虚霍金辐射交换如何给质量为 $\ll m_{\rm Pl}$ 的基本粒子被施瓦西黑洞非弹性散射带来新的可计算特征，该效应在 $(E_{\rm CM}/m_{\rm Pl})^2$ 展开中所处的阶，与引力子圈导致的量子引力领头阶散射量子修正 (这类修正最早在 [11,33,34] 中研究) 同阶。

My hope is that this review will give the reader a sense of how the tower of EFTs depicted in Fig. 1 gives a complete description of compact binaries in the perturbative regime, including all effects between the size of the compact objects themselves up to the scale of the radiation. It is beyond the scope here to provide full technical details of the calculations that have been performed using the EFT. There exist already several review articles that treat the various technical aspects in more details; see [42, 53, 54, 83, 98] . Finally, I apologize in advance that space limitations prevent me from giving here a complete guide to the vast literature on the subject. For example, not discussed here at all are the very recent applications of the EFT to the calculation of PM scattering observables, a topic that has drawn together the scattering amplitude, effective field theory, and traditional general relativity communities. A review of this rapidly developing subject can be found in [14].

我希望本综述能让读者感受到，图 1 所示的这套有效场论层级如何完整描述微扰区域的致密双星，包含从致密天体自身尺度到辐射尺度之间的全部效应。本文 scope 并不涵盖给出利用该 EFT 完成的计算的完整技术细节。目前已有多篇综述对各类技术细节做了更详尽的讨论, 参见 [42, 53, 54, 83, 98] 。最后，由于篇幅限制，我无法在这里对该方向的海量文献给出完整梳理，在此提前致歉。例如，本文完全没有讨论有效场论最近应用于 PM 散射观测量计算的工作，该方向结合了散射振幅、有效场论和传统广义相对论学界的研究。关于这一快速发展领域的综述可以参见 [14]。

Similarly I refer to the Snowmass 2021 article [61], which provides an up-to-date and exhaustive compilation of references on EFTs of gravity and their application to gravitational wave sources.

同样，我推荐读者参考 Snowmass 2021 会议文章 [61]，其中对引力有效场论及其在引力波源方向应用的相关文献做了最新的全面汇总整理。

Cross-references A general review on the description of low energy quantum gravity as an effective quantum field theory can be found in the contribution of J. Donoghue to this volume. See also the review articles [15, 35]. The Chap. 3, ”Effective Field Theory and Applications” provides a detailed description of the Feynman rules for perturbative gravity expanded about flat spacetime (see also the section Perturbative Quantum Gravity edited by I. Shapiro). The contribution by C. P. Burgess et al. has intellectual overlap with the discussion of dissipation and black hole horizons in section ”Event Horizon Dynamics in ZSOU”

交叉引用: 关于将低能量子引力描述为有效量子场论的综述, J. Donoghue 在本卷中的撰稿已有介绍, 另可参见综述文章 [15, 35]。第 3 章“有效场论及其应用”详细给出了平直背景下展开的微扰引力的费曼规则 (也可参见由 I. Shapiro 主编的微扰量子引力章节)。C. P. Burgess 等人的撰稿与“ZSOU 区的事件视界动力学”章节中关于耗散和黑洞视界的讨论存在内容重叠。

The One-Body Sector

单体扇区

We begin by constructing an EFT that characterizes the low-frequency response of an isolated compact astrophysical object to external gravitational perturbations. To that end, we imagine that we start first with the system in isolation. The details of its internal shape or composition depend sensitively on the microscopic theory. We assume in this review that the microscopic theory consists of GR coupled to the standard model (SM) of strong plus electroweak interactions. In this case, the compact object is, by definition, just some complicated many-body equilibrium state $\hat{\rho}$ with average total (ADM [6]) energy M , angular momentum J , and either zero (a black hole (We assume black holes with $Q_{em} = 0$ in this review.)) or very large baryon number. Although the focus in on the SM, the methods that we introduce in this review have been generalized (see [61] for a complete set of references) to include possible extensions of the SM that carry additional (e.g., dark matter) fields and therefore a richer zoology of compact stellar objects.

我们首先构造有效场论 (EFT)，来描述孤立致密天体对外部引力扰动的低频响应。为此，我们先从孤立系统出发。天体内部形状与组分的细节高度依赖微观理论。在本综述中，我们假设微观理论由广义相对论 (GR) 耦合强相互作用与电弱相互作用的标准模型 (SM) 构成。在这种情况下，根据定义，致密天体就是一个复杂的多体平衡态 $\hat{\rho}$，其平均总 (ADM [6]) 能量为 M，角动量为 J，重子数要么为零 (即黑洞，本综述中假设黑洞带有 $Q_{em} = 0$)，要么重子数极大。尽管我们重点关注标准模型，本综述介绍的方法已经被推广 (完整参考文献见 [61])，可涵盖标准模型的各类可能扩展，这类扩展引入额外 (例如暗物质) 场，因此包含更丰富的致密天体类型。

Regardless of the details of the internal structure, any type of self-gravitating distribution of matter will appear at long distances much larger than its radius $\mathcal{R}$ to be approximately point-like, with a well-defined ”center-of-mass” worldline $x^{\mu}(s)$. For example, the long-distance gravitational field of the isolated object has the same universal form $\sim G_N M/|\mathbf{x}|$ at spatial infinity, indistinguishable from a static point particle at the origin of the coordinate system. By going to large but finite distance, the gravitational field also encodes the angular momentum J as well as other multipole moments which do depend on the precise microscopic state $\hat{\rho}$. From the point of view of distant observers, these can also be described in terms of degrees of freedom localized on the defect worldline $x^{\mu}(s)$ (see sections "Radiative Corrections and TSP IR" and "Event Horizon Dynamics in TSP UV").

> 无论内部结构的细节如何，任何类型的自引力物质分布，在远大于其半径 $\mathcal{R}$ 的长距离下都会近似为类点，拥有明确定义的" 质心" 世界线 $x^{\mu}(s)$ 。例如，这个孤立天体的长距离引力场在空间无穷远具有相同的普适形式 $\sim G_N M/|\mathbf{x}|$ ，与坐标原点处的静态点粒子无法区分。在大但有限的距离处，引力场还包含角动量 J 以及其他依赖于具体微观状态 $\hat{\rho}$ 的多极矩。在远处观测者看来，这些量同样可以用缺陷世界线 $x^{\mu}(s)$ 上定域化的自由度描述 (参见章节" 辐射修正与 TSP 红外" 和"TSP 紫外中的事件视界动力学")。

Next, we consider how the equilibrium state of the compact object responds to external gravitational perturbations. Physically, we might imagine these perturbations to correspond to, e.g., on-shell gravitons coming in from past null infinity $\mathcal{I}^-$ and scattering off the object out to future null infinity $\mathcal{I}^+$, or perhaps to massive particles incoming from past timelike infinity i^- that get caught in the object' s gravitational field and generate, via off-shell exchange, tidal deformations of its shape. Within the point particle description, we can think of such probes generically as if we were turning on some gravitational field $g_{\mu\nu}$ with the appropriate boundary conditions at infinity, which interacts with the compact object. As long as the curvature length scale $\mathcal{L}$ associated with $g_{\mu\nu}$ is large compared to the size of the object $\sim \mathcal{R}$, we can continue to describe the response of the object systematically within a worldline EFT, as an expansion in powers in $G_N M/\mathcal{R} \ll 1$ and $L/\mathcal{R} \ll 1$.

> 接下来，我们考虑致密天体的平衡态如何响应外部引力扰动。物理上，我们可以认为这些扰动对应例如，从过去类空无穷远 $\mathcal{I}^-$ 入射的在壳引力子，被天体散射后向未来类空无穷远 $\mathcal{I}^+$ 出射；或是从过去类时无穷远 i^- 入射的大质量粒子，被天体的引力场捕获，通过离壳交换产生天体形状的潮汐形变。在点粒子描述中，我们可以将这类探针一般地视为，我们引入了某个满足无穷远合适边界条件的引力场 $g_{\mu\nu}$ ，它与致密天体相互作用。只要和 $g_{\mu\nu}$ 关联的曲率长度标度 $\mathcal{L}$ 远大于天体自身的尺寸 $\sim \mathcal{R}$ ，我们就可以继续在世界线 EFT 框架内系统地描述天体的响应，将其按 $G_N M/\mathcal{R} \ll 1$ 和 $L/\mathcal{R} \ll 1$ 的幂次展开。

In this section, we assume for simplicity that the internal dynamics of the compact object is "gapped" at some frequency scale much larger than the scale $1/\mathcal{L}$ set by the curvature. In this limit, dissipation, e.g., the possibility of absorption of gravitational energy-momentum by the object, is suppressed. We will come back to the inclusion of such non-conservative effects later on in section "Event Horizon Dynamics in YSOUV, Thus, by assumption, the relevant degrees of freedom in the IR consist of:

> 在本节中，为简化起见我们假设致密天体的内部动力学在远大于曲率设定的标度 $1/\mathcal{L}$ 的某个频率标度处存在" 能隙"。在这个极限下，耗散 (例如天体吸收引力能量动量的可能性) 被压低。我们会在之后"YSOUV 中的事件视界动力学" 章节回到这类非守恒效应的讨论，因此根据假设，红外区域的相关自由度为:

- The spacetime metric $g_{\mu\nu}(x)$.

> - 时空度规 $g_{\mu\nu}(x)$ 。

- A worldline $x^\mu(s)$, describing the center-of-mass motion of the object.

> - 一条世界线 $x^\mu(s)$ ，描述天体的质心运动。

- A local Lorentz frame $e^\mu{}_a(s)$

> - 一个局域洛伦兹系 $e^\mu{}_a(s)$

$$g_{\mu\upsilon}(x(s))\, e^\mu{}_a e^\upsilon{}_b = \eta_{ab}, \eta^{ab} e^\mu{}_a e^\upsilon{}_b = g_{\mu\upsilon}(x(s)),$$

at each point $x^\mu(s)$ along the worldline. This frame describes the orientation of the compact object relative to distant observers; in particular the object's rotational velocity is

> 位于世界线 $x^\mu(s)$ 上的每一点。这个参考系描述了致密天体相对于远处观测者的取向；特别地，天体的转动速度为

$$\Omega^{ab} = g^{\mu\upsilon} e^a{}_\mu \left(\dot{x}^\rho \nabla_\rho\right) e^b{}_\upsilon = -\Omega^{ba}. \tag{2}$$

The local frame $e^\mu{}_a(s)$ is necessary to describe objects with non-zero spin. In the absence of gravity, it was introduced by Regge and Hanson [66] to treat the classical motion of relativistic spinning particles coupled to electromagnetic fields. In flat spacetime, the worldline degrees of freedom $(x^\mu, e^\mu{}_a)$ parameterize points on the Poincare group of isometries, which are in general spontaneously broken by the presence of the compact object. The extension of the Regge-Hanson formalism to curved spacetime and its applications to perturbative binary dynamics first appeared in [95]. A related treatment of spinning particles from the point of view of nonlinearly realized symmetries and Goldstone's theorem can be found in Ref. [29]. While the inclusion of spin effects is crucial for constructing gravitational wave templates relevant to phenomenology, for space reasons we will omit any detailed discussion in this review article. Up-to-date reviews of spin effects in the worldline EFT can be found in Refs. [83, 98].

> 描述非零自旋物体需要用到局部坐标系 $e^\mu{}_a(s)$ 。在无引力的情况下，该坐标系由雷格和汉森引入 [66],用于处理与电磁场耦合的相对论性自旋粒子的经典运动。在平直时空中,世界线自由度 $(x^\mu, e^\mu{}_a)$ 对庞加莱等距群上的点进行参数化，一般而言该对称性会因致密天体的存在而自发破缺。将雷格-汉森形式推广到弯曲时空，并将其应用于微扰双星动力学的工作最早发表在文献 [95]。从非线性实现对称性和戈德斯通定理的角度对自旋粒子的相关处理可以参见文献 [29]。尽管纳入自旋效应对构建符合唯象学要求的引力波模板至关重要，由于篇幅限制我们不会在这篇综述中展开详细讨论。关于世界线有效场论中自旋效应的最新综述可以参见文献 [83, 98]。

There is a gauge redundancy in the variables $\left(g_{\mu\nu}, x^\mu(s), e^a{}_\mu(s)\right)$ that define the EFT, namely:

> 定义有效场论的变量 $\left(g_{\mu\nu}, x^\mu(s), e^a{}_\mu(s)\right)$ 中存在规范冗余，即:

1. Spacetime diffeomorphism invariance: $\delta x^\mu = \xi^\mu(x), \delta g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}$, $\delta e^\mu{}_a(s) = [\xi, e_a]^\mu(x(s))$, etc.

> 1. 时空微分同胚不变性: $\delta x^\mu = \xi^\mu(x), \delta g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}$ 、$\delta e^\mu{}_a(s) = [\xi, e_a]^\mu(x(s))$ 等。

2. Reparameterizations $s \to \tilde{s} = \tilde{s}(s)$ of the worldline time coordinate.

2. 世界线时间坐标的重参数化 $s \to \tilde{s} = \tilde{s}(s)$。

We assume that there are smooth limits $g_{\mu\nu} \to \eta_{\mu\nu}$ or $\mathcal{R} \to 0$. We also assume that Wilsonian decoupling of UV physics holds, guaranteeing that whatever the microscopic description, the low-frequency response of the isolated compact object is described by a local effective theory of the generic form

我们假设存在光滑极限 $g_{\mu\nu} \to \eta_{\mu\nu}$ 或 $\mathcal{R} \to 0$。我们还假设威尔逊紫外物理退耦成立，这保证了无论微观描述如何，孤立致密天体的低频响应都可以用如下一般形式的局域有效理论描述

$$S = S_{EH}\left[g_{\mu\nu}\right] + S_{pp}\left[g_{\mu\nu}, x^{\mu}(s), e^{\mu}{}_{a}(s)\right], \tag{3}$$

where S_{EH} is the bulk gravity theory Eq. (1) and $S_{pp}[g, x(s), e(s)]$ is a term localized on the worldline. It is an infinite sum of gauge invariants constructed from the spacetime curvature and its derivatives, with Wilson coefficients that, by dimensional analysis, scale as successively larger powers of the radius $\mathcal{R}$.

其中 S_{EH} 是体引力理论式 (1)，$S_{pp}[g, x(s), e(s)]$ 是定域在世界线上的项。它是由时空曲率及其导数构造的规范不变量的无穷和，通过量纲分析可知，其威尔逊系数的标度对应半径 $\mathcal{R}$ 依次升高的幂次。

The Wilson coefficients in S_{pp} are free parameters from the point of view of the low energy theory, to be determined by matching to the UV theory, as we describe in more detail below. However, even if the microscopic description is unknown, the EFT still has predictive power. Since spacetime or worldline derivatives are small, $\partial_\mu, d/ds \ll \mathcal{R}^{-1}, S_{pp}$ may be truncated at a fixed order in the derivative expansion at the expense of introducing (presumably small) errors suppressed by powers of $\mathcal{R}/L \ll 1$. Therefore, to calculate an observable to finite precision in the EFT, one only needs to know a finite set of Wilson coefficients, which can be regarded as experimental inputs.

从低能理论的角度来看，S_{pp} 中的威尔逊系数是自由参数，需要通过匹配紫外理论来确定，我们会在下文更详细地说明。但即使微观描述未知，该有效场论仍具有预言能力。由于时空或世界线导数很小，$\partial_\mu, d/ds \ll \mathcal{R}^{-1}, S_{pp}$ 可以在导数展开中固定阶数处截断，代价是引入被 $\mathcal{R}/L \ll 1$ 的幂次压低的 (通常很小) 误差。因此，要在有效场论中以有限精度计算可观测量，我们只需要知道有限组威尔逊系数，这些系数可以看作实验输入。

The expansion of S_{pp} up to second order in spacetime derivatives takes the form

S_{pp} 在时空导数下展开到二阶的形式为

$$S_{pp} = S_{pp}^{(0)} + S_{pp}^{(1)} + S_{pp}^{(2)} + \cdots, \tag{4}$$

where the unique leading (zero derivative) term is proportional to the proper time $d\tau = \sqrt{g_{\mu\nu}dx^{\mu}dx^{\nu}}$ elapsed along the worldline

其中唯一的领头阶 (零导数) 项正比于沿世界线经过的固有时 $d\tau = \sqrt{g_{\mu\nu}dx^\mu dx^\nu}$

$$S_{pp}^{(0)} = -m\int d\tau \tag{5}$$

with $m > 0$ a real parameter with dimensions of mass. In the $\mathcal{R} \to 0$ limit, we ignore the backreaction of the compact object on the spacetime metric, in which case the parameter m becomes irrelevant and the leading order equations of motion for x^μ is simply the geodesic equation

其中 $m > 0$ 是一个量纲为质量的实参数。在 $\mathcal{R} \to 0$ 极限下，我们忽略致密天体对时空度规的反作用，此时参数 m 变得无关紧要，x^μ 的领头阶运动方程就是测地线方程

$$\delta S_{pp}^{(0)} = 0 \Rightarrow a^\mu = \frac{dx^\upsilon}{d\tau}\nabla_\upsilon \frac{dx^\mu}{d\tau} = 0, \tag{6}$$

as expected on the basis of the Einstein equivalence principle.

这符合爱因斯坦等效原理的预期。

On the other hand, when we include two-body interactions in section ”Perturbative Binary Dynamics,” we will need to account for the gravitational field sourced by the compact object, in which case the parameter m does not drop out of the dynamics. To fix the precise dependence of the EFT mass parameter m on the microscopic properties of the compact, we must perform a matching calculation to the full UV theory. This is accomplished by computing the same physical observable in both the EFT and in the UV theory, adjusting the EFT parameters so that both results agree in the overlapping regime of validity of the two theories.

另一方面，当我们在“微扰双星动力学”章节中纳入两体相互作用时，就必须考虑致密天体自身产生的引力场，此时参数 m 不会从动力学中消失。为了确定有效场论 (EFT) 质量参数 m 对致密天体微观性质的精确依赖关系，我们必须对完整紫外理论做匹配计算: 只需分别在 EFT 和紫外理论中计算同一个物理可观测量，调整 EFT 参数使得两个结果在两个理论的重叠适用区间内一致即可。

For the case of the mass parameter m , a convenient quantity to match is simply the graviton one-point function sourced by the compact object, probed by observers at spatial infinity. In full general relativity, we choose deDonder coordinates x^μ such that the metric takes the form

对于质量参数 m ，一个方便的匹配量就是空间无穷远观测者探测到的、由该致密天体产生的引力子单点函数。在完整广义相对论中，我们选取德唐德坐标 x^μ ，此时度规的形式为

$$g_{\mu\upsilon}(x) = \eta_{\mu\upsilon} + h_{\mu\upsilon}(x), \partial_\upsilon h^{\mu\upsilon} = \frac{1}{2}\partial^\upsilon h, \tag{7}$$

$(h = h^\rho{}_\rho = \eta^{\rho\sigma}h_{\rho\sigma})$ everywhere on spacetime. The graviton field $h_{\mu\nu}(x)$ need not be small, but, by the Einstein equations, it falls off at long distances from the compact object [115]. In particular, at $|\mathbf{x}| \to \infty$, the gravitational field of the compact object becomes

$(h = h^\rho{}_\rho = \eta^{\rho\sigma}h_{\rho\sigma})$ 在全时空成立。引力子场 $h_{\mu\nu}(x)$ 不必是小量，但根据爱因斯坦方程，它在远离致密天体的长距离处衰减 [115]。特别地，在 $|\mathbf{x}| \to \infty$ 处，致密天体的引力场为

$$\lim_{r\to\infty} h_{\mu\upsilon}^{Full}(x) = \frac{2G_N M_{ADM}}{|\mathbf{x}|}\left(\eta_{\mu\upsilon} + u_\mu u_\upsilon\right) \tag{8}$$

in the center-of-mass frame (CM), in which the object's four-momentum is

在质心系 (CM) 中，天体的四动量为

$$P^\mu = (M_{ADM}, 0, 0, 0) \equiv M_{ADM} u^\mu, \tag{9}$$

where M_{ADM} is the ADM mass of the compact object.

其中 M_{ADM} 是该致密天体的 ADM 质量。

In the EFT, we instead solve the deDonder gauge linearized Einstein equations, taking the source term to be

在 EFT 中，我们改为求解德唐德规范下的线性化爱因斯坦方程，取源项为

$$T_{pp}^{\mu\upsilon} = \frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\upsilon}} S_{pp}^{(0)} = m u^\mu u^\upsilon \delta^3(\mathbf{x}), \tag{10}$$

which corresponds to a point defect at rest at the origin. This yields the result

这对应原点处静止的点缺陷，得到结果

$$h_{\mu\upsilon}^{EFT}(x) = -\frac{i}{2m_{\rm Pl}^2}\int d^4x' D_R(x-x') P_{\mu\upsilon;\rho\sigma} T_{pp}^{\rho\sigma}(x') \approx \frac{2G_N m}{|\mathbf{x}|}\left(\eta_{\mu\upsilon} + u_\mu u_\upsilon\right),$$

(11)

where the graviton propagator in deDonder gauge is a product of the massless retarded Green's function

其中德唐德规范下的引力子传播子是无质量推迟格林函数的乘积

$$-iD_R(x) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot x}}{(k^0 + i0^+)^2 - \mathbf{k}^2} \tag{12}$$

and the tensor structure is $P_{\mu\nu;\rho\sigma} = \frac{1}{2}\left[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}\right]$ in four spacetime dimensions. Equation (11) is the gravitational field in the EFT, valid up to corrections from higher derivative operators in S_{pp} that fall off like more powers of $1/r$.

其张量结构在四维时空下为 $P_{\mu\nu;\rho\sigma} = \frac{1}{2}\left[\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}\right]$。式 (11) 是 EFT 中的引力场，适用范围包含 S_{pp} 中更高导数算符带来的修正，这些修正随 $1/r$ 的更高次幂衰减。

Because the full GR and EFT calculations are performed in the same coordinate system, in this example, matching is just the statement that, at long distances, $|\mathbf{x}| \to \infty$, where both descriptions are equally valid, $h_{\mu\nu}^{Full} = h_{\mu\nu}^{EFT}$, which yields

由于完整广义相对论和 EFT 的计算在同一坐标系下进行，在本例中，匹配只需满足: 在长距离 $|\mathbf{x}| \to \infty$ 处 (两种描述在此同样适用)，$h_{\mu\nu}^{Full} = h_{\mu\nu}^{EFT}$ 成立，由此得到

$$m = M_{ADM}. \tag{13}$$

We conclude that in EFT, the point particle mass should be identified with the ADM mass of the compact object in isolation, i.e., not subject to external fields. This is of course what we would intuitively expect, but the point of this (perhaps somewhat pedantic) exercise is to illustrate that there is a systematic procedure for relating the physical properties of the compact object to the parameters of its worldline proxy in the EFT.

我们得出结论:EFT 中的点粒子质量应当对应孤立 (即不受外场影响) 致密天体的 ADM 质量。这当然符合我们的直觉预期，但这个 (或许有些学究气的) 练习的意义在于说明，存在一套系统方法，可以将致密天体的物理性质与其在 EFT 中的世界线近似参数联系起来。

The matching condition Eq. (13) can in principle receive corrections on the EFT side in powers of G_N, e.g., from the self-energy of the static field sourced by the worldline. In the EFT, such effects in general carry short-distance singularities and depend on the choice of UV regulator, e.g., dimensional regularization sets such power UV divergent contributions to zero. While the UV behavior of the EFT differs from that in the full theory, any discrepancy can be compensated by renormalizing the local counterterms (Wilson coefficients) in S_{pp} order-by-order in perturbation theory.

原则上，式 (13) 的匹配条件在 EFT 侧会收到 G_N 幂次的修正，例如来自世界线产生的静态场的自能贡献。在 EFT 中，这类效应一般带有短距离奇点，依赖于紫外正则化的选择: 比如维数正则化就会将这类幂次紫外发散贡献置零。虽然 EFT 的紫外行为和完整理论不同，但任何差异都可以通过微扰论逐阶重整化 S_{pp} 中的局域抵消项 (威尔逊系数) 来补偿。

Because the leading order term in S_{pp} is universal, to resolve the internal structure of the object, we need to keep the higher derivative terms. For compact objects, with $\kappa \gtrsim \mathcal{O}(1)$, we expect that by dimensional analysis, the Wilson coefficients of terms with n-derivatives should scale as $m\mathcal{R}^n$ up to order unity numerical factors. To simplify the discussion, I will assume in the rest of this section that the compact object does not carry any permanent multipole moments, encoded in the EFT as local Lorentz tensors on the worldline, in its equilibrium state. For example, a Kerr black hole is characterized by an infinite tower of multipoles, all proportional to powers of its spin [65], which imply a rich structure of worldline couplings that we ignore here; see [83] for a detailed review of such effects.

由于 S_{pp} 中的领头阶项是普适的，若要分辨天体的内部结构，我们需要保留高阶导数项。对于满足 $\kappa \gtrsim \mathcal{O}(1)$ 的致密天体，我们可以通过量纲分析预期，带有 n 阶导数项的威尔逊系数，在包含数值阶幺因子的前提下，标度规律为 $m\mathcal{R}^n$。为简化讨论，我将在本节剩余部分假设: 致密天体在平衡态不携带任何永久多极矩，多极矩在有效场论中被编码为世界线上的局域洛伦兹张量。例如，克尔黑洞的特征是无穷多极矩序列，所有极矩都和它的自转角幂次成正比 [65]，这意味着丰富的世界线耦合结构，我们在此处略去；相关效应的详细综述可见 [83]。

To construct the full set of invariants at a given order in the derivative expansion, one may use the equations of motion to eliminate ”redundant” terms [48]. For example, the zeroth-order worldline equations of motion imply that $a^\mu = 0$, so that terms involving the acceleration can be omitted from S_{pp}. Similarly, the leading Einstein equations imply that the Ricci curvature $R_{\mu\nu}$ can be traded for a contact term localized on the particle worldline:

为了构造导数展开中给定阶数的全套不变量，可以利用运动方程消去「冗余」项 [48]。例如，零阶世界线运动方程给出 $a^{\mu}=0$ ，因此包含加速度的项可以从 S_{pp} 中省略。同理，领头阶爱因斯坦方程指出，里奇曲率 $R_{\mu\nu}$ 可以替换为局域在粒子世界线上的接触项:

$$R_{\mu\nu}(x)=8\pi G_N m\int d\tau\frac{\delta^4(x-x(\tau))}{\sqrt{g}}\left(u^{\mu}u^{\upsilon}-\frac{1}{2}g^{\mu\upsilon}\right). \tag{14}$$

As consequence of the equations of motion, we may therefore assume that $S_{pp}^{(1)}=0$, while at the two-derivative level, we can drop operators constructed out of the Ricci curvature, e.g.,

因此作为运动方程的结果，我们可以认为 $S_{pp}^{(1)}=0$ ，而在两导数阶，我们可以丢弃由里奇曲率构造的算符，例如

$$\int d\tau R(x(\tau)),\int d\tau u^{\mu}u^{\upsilon}R_{\mu\upsilon}(x(\tau)), \tag{15}$$

since by Eq. (14), these can be absorbed into the definition of the EFT parameter m . Thus by performing field redefinitions, which have no effect on the physical predictions, we can assume that $S_{pp}^{(2)}=0$ as well.

根据式 (14)，这些都可以被吸收到有效场论参数 m 的定义中。因此，通过进行不影响物理预言的场重定义，我们还可以得到 $S_{pp}^{(2)}=0$ 也成立。

The first genuinely physical finite size corrections only appear at the fourth derivative order. Using the algebraic properties of the Riemann tensor in four spacetime dimensions, the most general structure consistent with our assumptions can be expressed as

第一个真正意义上的物理有限大小修正仅出现在四导数阶。利用四维时空下黎曼张量的代数性质，符合我们假设的最一般结构可以写为

$$S_{pp}^{(4)}=c_E m\mathcal{R}^4\int d\tau E_{\mu\upsilon}E^{\mu\upsilon}+c_B m\mathcal{R}^4\int d\tau B_{\mu\upsilon}B^{\mu\upsilon}+c_{EB}m\mathcal{R}^4\int d\tau E_{\mu\upsilon}B^{\mu\upsilon}$$

(16)

for objects that do not carry spin. In this equation, $E_{\mu\nu}$ and $B_{\mu\nu}$ are the ”electric” and ”magnetic” components of the Weyl tensor relative to the particle worldline

适用于不带自旋的天体。在此式中，$E_{\mu\nu}$ 和 $B_{\mu\nu}$ 是外尔张量相对于粒子世界线的「电」分量和「磁」分量

$$E_{\mu\nu}=W_{\mu\rho\nu\sigma}u^{\rho}u^{\sigma}, \tag{17}$$

$$B_{\mu\upsilon}=\widetilde{W}_{\mu\rho\upsilon\sigma}u^{\rho}u^{\sigma}, \tag{18}$$

where $\widetilde{W}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}W^{\alpha\beta}{}_{\rho\sigma}$ is the dual curvature. (We will use the terms "Weyl" and "Riemann" interchangeably in light of the fact that they are equivalent on-shell.) Using the algebraic properties of the Weyl tensor, one can show that $E_{\mu\nu}$ and $B_{\mu\nu}$ are symmetric, traceless $E^{\mu}{}_{\mu} = B^{\mu}{}_{\mu} = 0$ and transverse to the velocity $E_{\mu\upsilon}u^{\upsilon} = B_{\mu\upsilon}u^{\upsilon} = 0$, so that they indeed encode all ten independent components of the Weyl tensor.

其中 $\widetilde{W}_{\mu\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}W^{\alpha\beta}{}_{\rho\sigma}$ 是对偶曲率。(由于二者在在壳条件下等价，我们会交叉使用「外尔」和「黎曼」这两个术语。) 利用外尔张量的代数性质可以证明，$E_{\mu\nu}$ 和 $B_{\mu\nu}$ 是对称、无迹的 $E^{\mu}{}_{\mu} = B^{\mu}{}_{\mu} = 0$ ，且正交于速度 $E_{\mu\upsilon}u^{\upsilon} = B_{\mu\upsilon}u^{\upsilon} = 0$ ，因此它们确实编码了外尔张量全部十个独立分量。

We have defined the dimensionless Wilson coefficients in Eq. (16) in such a way that, for relativistic compact objects $c_E \sim c_B \sim \mathcal{O}(1)$. On the other hand, it is possible to define a parity transformation, such that any given instant τ in a comoving frame $u^{\hat{\mu}} = (1,0,0,0)$ centered at $x^{\hat{\mu}}(\tau) \equiv 0$ acts as

我们在式 (16) 中定义无量纲威尔逊系数时，已经将其整理为适用于相对论性致密天体 $c_E \sim c_B \sim \mathcal{O}(1)$ 的形式。另一方面，我们可以定义宇称变换，使得任意时刻，共动坐标系 $u^{\hat{\mu}} = (1,0,0,0)$ (原点在 $x^{\hat{\mu}}(\tau) \equiv 0$) 中的 τ 满足变换关系

$$P : E_{\hat{i}\hat{j}}(0) \to +E_{\hat{i}\hat{j}}(0) , B_{\hat{i}\hat{j}}(0) \to -B_{\hat{i}\hat{j}}(0) . \tag{19}$$

This first two terms in Eq. (16) are even under P , while the last term is parity odd. For neutron stars, whose microscopic properties are well described by the physics of the SM, parity violating effects are expected to be highly suppressed, $\left|c_{EB}^{NS}\right| \ll 1$. On the other hand, more exotic compact objects predicted by extensions of the SM, e.g., containing a "dark sector" with sizeable parity violation, may be characterized by a Wilson coefficient $c_{EB} \sim \mathcal{O}(1)$; see [86].

式 (16) 的前两项在 P 变换下为偶，最后一项为宇称奇。对于微观性质已被标准模型物理很好描述的中子星来说，宇称破坏效应预计会被高度抑制，$\left|c_{EB}^{NS}\right| \ll 1$ 。另一方面，标准模型扩展所预言的更奇异致密天体，例如包含存在显著宇称破坏的“暗区”，其特征可以用威尔逊系数 $c_{EB} \sim \mathcal{O}(1)$ 描述；参见文献 [86]。

If the coefficients in Eq. (16) are non-zero, the compact's object motion is no longer geodesic. In fact $c_{E,B} \neq 0$ characterize the static tidal quadrupolar response of the compact object to external gravitational fields. We can see this intuitively, by putting a small "moon" in orbit about the compact object, corresponding to a weak external Newtonian potential $\Phi_{\text{ext}}(\mathbf{x}, t)$. This external potential induces tides on the object, distorting its shape away from perfect spherical symmetry. This deformation can then be detected by far away inertial observers, by measuring the long-distance gravitational field that it produces.

如果式 (16) 中的系数非零，致密天体的运动不再是测地线。实际上 $c_{E,B} \neq 0$ 描述了致密天体对外引力场的静态潮汐四极响应。我们可以直观理解这一点: 将一颗小“卫星”置于致密天体的轨道上，对应一个弱的外部牛顿势 $\Phi_{\text{ext}}(\mathbf{x}, t)$ 。此外部势会在天体上激发出潮汐，使其形状偏离完美球对称。远处的惯性观测者可以通过测量该形变产生的长程引力场探测到这一变化。

We write the total Newtonian gravitational potential as

我们将总牛顿引力势写为

$$\Phi = \Phi_{\mathrm{co}} + \Phi_{\mathrm{ext}} + \delta\Phi, \tag{20}$$

where $\Phi_{\mathrm{co}} = -G_N m/|\mathbf{x}|$ is the unperturbed long-distance field produced by the compact object and $\delta\Phi$ is the response, i.e., the potential generated by the tidal bulge generated by the moon orbiting the object. Taking the compact object to be at rest at $\mathbf{x} = 0$, and that the moon' s orbit is non-relativistic, with velocity $v \ll 1$, we have

其中 $\Phi_{\mathrm{co}} = -G_N m/|\mathbf{x}|$ 是致密天体产生的未受扰长程场，$\delta\Phi$ 是响应，即绕天体运行的卫星激发的潮汐鼓胀产生的势。假设致密天体静止于 $\mathbf{x} = 0$ ，且卫星轨道是非相对论性的，速度为 $v \ll 1$ ，我们得到

$$E_{ij} \approx W_{0i0j} = -\partial_i\partial_j\Phi + \cdots \tag{21}$$

and $B_{ij} \sim \mathcal{O}(v) \cdot E_{ij}$, so that, in the linear response approximation,

且 $B_{ij} \sim \mathcal{O}(v) \cdot E_{ij}$ ，因此在线性响应近似下，

$$S_{pp}^{(4)} \approx m\mathcal{R}^4 c_E \int dt \big(\partial_i\partial_j\Phi(0,t)\big)^2 \supset 2m\mathcal{R}^4 c_E \int dt\left(\partial_i\partial_j\Phi_{\mathrm{ext}}(0,t)\right)\partial^i\partial^j\delta\Phi(0,t) + \cdots$$

(22)

We have dropped a (UV divergent) self-energy term induced by the insertion of $\Phi_{\mathrm{co}}(\mathbf{x}=0)$ into $S_{pp}^{(4)}$, which can be absorbed into the definition of m , as well as a term of order $(\delta\Phi)^2$ which is assumed to be small.

我们已经舍去了由将 $\Phi_{\mathrm{co}}(\mathbf{x}=0)$ 插入 $S_{pp}^{(4)}$ 诱导的 (紫外发散的) 自能项，该项可以被吸收到 m 的定义中，同时舍去了量级为 $(\delta\Phi)^2$ 的项，该假定为小量。

To observers far away, the coupling to the induced field has the form of the Newtonian gravitational interaction

对于远处的观测者，与感应场的耦合具有牛顿引力相互作用的形式

$$\mathcal{L}_{\mathrm{Newt}} = -\rho_{\ell=2}(\mathbf{x},t)\,\delta\Phi, \tag{23}$$

sourced by a pointlike mass quadrupole distribution located at the origin

由位于原点的点质量四极分布作为源

$$\rho_{\ell=2}(\mathbf{x},t) = \frac{1}{2}I^{ij}(t)\,\partial_i\partial_j\delta^3(\mathbf{x}), \tag{24}$$

which we can read off Eq. (22)

我们可以从式 (22) 直接得到

$$I^{ij}(t) = 4c_E m\mathcal{R}^4 E_{\mathrm{ext}}^{ij}(0,t) \tag{25}$$

$E^{ij}_{\text{ext}} = -\partial^i \partial^j \Phi_{\text{ext}}$. The induced quadrupole moment has the precise form that one would expect for the linear gravitational perturbation of a spherically symmetric distribution whose internal dynamics is gapped, so that the response to slowly varying external fields is instantaneous and linearly proportional to the perturbing field.

> $E^{ij}_{\text{ext}} = -\partial^i \partial^j \Phi_{\text{ext}}$ 。感应四极矩的形式与我们对内部动力学有能隙的球对称分布的线性引力扰动的预期完全一致，因此对缓变外场的响应是瞬时的，且与扰动场线性成正比。

For a nearly static and weakly self-gravitating Newtonian mass distribution, it is conventional for historical reasons to denote the constant of proportionality between the external tidal field $\partial_i \partial_j \Phi_{\text{ext}}$ and the induced mass quadrupole as the ($\ell = 2$) static Love number $k_{\ell=2}$ of the system (following the definition in [10]):

> 对于近静态、弱自引力的牛顿质量分布，出于历史原因，习惯上将外潮汐场 $\partial_i \partial_j \Phi_{\text{ext}}$ 与感应质量四极之间的比例常数记为系统的 ($\ell = 2$) 静态勒夫数 $k_{\ell=2}$ (遵循文献 [10] 中的定义):

$$I^{ij}(t) = \frac{2}{3} \frac{\mathcal{R}^5}{G_N} k_{\ell=2} E^{ij}_{\text{ext}}(t, 0) . \tag{26}$$

So by matching the linear response in the EFT to that predicted by Newtonian geo-elasticity theory (the full theory), we learn that the Wilson coefficient c_E describes the tidal susceptibility in the worldline EFT:

> 因此通过将有效场论中的线性响应与牛顿地球弹性理论 (完整理论) 预言的响应匹配，我们可知威尔逊系数 c_E 描述了世界线有效场论中的潮汐极化率:

$$c_E = \frac{1}{6} \left(\frac{\mathcal{R}}{G_N m} \right) k_{\ell=2}. \tag{27}$$

This interpretation of the linear response generalizes straightforwardly to the case of $\ell > 2$ induced mass multipole moments, as well as gravitomagnetic interactions, which in general induce ”current” multipole moments (i.e., moments of the angular momentum density) for each $\ell \geq 2$. In the worldline EFT, the static linear response of a fully relativistic spherically symmetric object is encoded in the higher-derivative interactions

> 这种线性响应的解释可以直接推广到 $\ell > 2$ 诱导质量多极矩的情况，也可以推广到引力磁相互作用，一般来说引力磁相互作用会对每个 $\ell \geq 2$ 诱导“流”多极矩 (即角动量密度的矩)。在世界线有效场论中，完全相对论性球对称天体的静态线性响应被编码在高阶导数相互作用中

$$m\mathcal{R}^{2\ell} \int d\tau \left(\nabla^{\perp \langle \alpha_1} \cdots \nabla^{\perp \alpha_{\ell-2} \rangle} E^{\mu\nu} \right) \left(\nabla^{\perp}_{\langle \alpha_1} \cdots \nabla^{\perp}_{\alpha_{\ell-2} \rangle} E_{\mu\nu} \right),$$

$$m\mathcal{R}^{2\ell} \int d\tau \left(\nabla^{\perp \langle \alpha_1} \cdots \nabla^{\perp \alpha_{\ell-2} \rangle} B^{\mu\nu} \right) \left(\nabla^{\perp}_{\langle \alpha_1} \cdots \nabla^{\perp}_{\alpha_{\ell-2} \rangle} B_{\mu\nu} \right),$$

$$m\mathcal{R}^{2\ell} \int d\tau \left(\nabla^{\perp \langle \alpha_1} \cdots \nabla^{\perp \alpha_{\ell-2} \rangle} E^{\mu\nu} \right) \left(\nabla^{\perp}_{\langle \alpha_1} \cdots \nabla^{\perp}_{\alpha_{\ell-2} \rangle} B_{\mu\nu} \right),$$

where the transverse covariant derivative is $\nabla^{\perp}_{\mu} = \nabla_{\mu} - u_{\mu} (u \cdot \nabla)$ and $\langle \cdots \rangle$ denotes the symmetric traceless projection of the enclosed indices.

其中横向协变导数为 $\nabla^{\perp}_{\mu} = \nabla_{\mu} - u_{\mu}(u \cdot \nabla)$，$\langle\cdots\rangle$ 表示包围指标的对称无迹投影。

Going beyond the static approximation, the linear response is also characterized by terms involving time derivatives $\dot{E}_{\mu\nu} = u^{\rho}\nabla_{\rho}E_{\mu\nu}$, or $\dot{B}_{\mu\nu}$, e.g., a term of the form

超出静态近似后，线性响应还以含时间导数 $\dot{E}_{\mu\nu} = u^{\rho}\nabla_{\rho}E_{\mu\nu}$ 或 $\dot{B}_{\mu\nu}$ 的项为特征，例如，一项形式为

$$m\mathcal{R}^6 \int d\tau \dot{E}_{\mu\nu}\dot{E}^{\mu\nu}$$

acts as a high-pass filter for electric perturbations. It is even also possible to incorporate effects beyond the linear response approximation. For instance, a term $m\mathcal{R}^6 \int d\tau E^{\mu}{}_{\nu}E^{\nu}{}_{\rho}E^{\rho}{}_{\mu}$ on the worldline reflects nonlinear couplings between gravity and the UV modes that have been integrated out of the full theory.

对电扰动起到高通滤波器的作用。我们甚至还可以纳入线性响应近似之外的效应。例如，世界线上的一项 $m\mathcal{R}^6 \int d\tau E^{\mu}{}_{\nu}E^{\nu}{}_{\rho}E^{\rho}{}_{\mu}$ 反映了引力与已从完整理论积分剔除的 UV 模式之间的非线耦合。

For fully relativistic compact objects which have strong internal gravity, the notion of induced multipole moments as in Eq. (26) in the full theory (GR) is not invariant under diffeomorphisms. Instead, the Love numbers are defined as the Wilson coefficients of an effective Lagrangian, i.e.,

对于具有强内部引力的完全相对论性致密天体，完整理论 (广义相对论) 中式 (26) 给出的诱导多极矩概念在微分同胚下不具有不变性。相反，勒夫数被定义为有效拉格朗日量的威尔逊系数，即

$$S^{\text{Love}}_{\ell=2} \equiv \frac{1}{6}k^{E}_{\ell=2}\left(\frac{\mathcal{R}^5}{G_N}\right)\int d\tau E_{\mu\nu}E^{\mu\nu} + \frac{1}{6}k^{B}_{\ell=2}\left(\frac{\mathcal{R}^5}{G_N}\right)\int d\tau B_{\mu\nu}B^{\mu\nu}, \tag{28}$$

and similarly on for $\ell > 2$. This definition has the advantage of being fully gauge invariant under coordinate transformations.

对 $\ell > 2$ 也有类似的定义。该定义的优势在于在坐标变换下完全规范不变。

Matching the Love numbers in the EFT Lagrangian to full GR is achieved by comparing gauge invariant observables. For this purpose, a conceptually clean (in principle) observable is the quantum mechanical probability amplitude for elastic $1 \to 1'$ elastic graviton scattering off the compact object [54]. One would calculate the amplitude in the full theory, by linearizing the Einstein equations around the background field $\bar{g}_{\mu\nu}$ sourced by the compact object, with asymptotic boundary conditions

将有效场论拉格朗日量中的勒夫数匹配到完整广义相对论可通过比较规范不变可观测量实现。为此，一个概念上 (原则上) 清晰的可观测量是弹性 $1 \to 1'$ 引力子弹性散射致密天体的量子力学概率幅 [54]。我们可以在完整理论中计算该振幅: 对致密天体源的背景场 $\bar{g}_{\mu\nu}$ 周围的爱因斯坦方程做线性化，结合渐近边界条件

$$\lim_{r\to\infty} h_{\mu\nu}(x) \to \varepsilon^{h}_{\mu\nu}(k)e^{-ik\cdot x} + \frac{\mathcal{A}_{\mu\nu}}{r}e^{-i\omega(t-r)}, \tag{29}$$

corresponding to an incoming plane wave with four-momentum k^μ and helicity h from past null infinity $\mathcal{I}^-$ and an outgoing (scattered) spherical wave at future infinity $\mathcal{I}^+$ with frequency $\omega = k^0$. Because the ingoing/outgoing waves have direct physical meaning as graviton asymptotic states, the scattering amplitude $\mathcal{A}_{\mu\nu}$ is gauge invariant and can be compared to the same quantity in the EFT.

该边界条件对应来自过去类空无穷远 $\mathcal{I}^-$ 、四动量为 k^μ 螺旋度为 h 的入射平面波，以及未来类空无穷远 $\mathcal{I}^+$ 处频率为 $\omega = k^0$ 的出射 (散射) 球面波。由于入/出射波作为引力子渐近态具有直接物理意义，散射振幅 $\mathcal{A}_{\mu\nu}$ 是规范不变的，可以与有效场论中的同一物理量做比较。

In the EFT, one calculates the amplitude $\mathcal{A}_{\mu\nu}$ for an incoming plane wave of definite helicity h using the Feynman rules from Eq. (16) expanded about flat space. In addition to terms where the graviton scatters off the mass monopole, m , there is a term in the scattering amplitude from the contact terms in Eq. (16), e.g., the electric Love operator contributes a term of the form

在有效场论中，我们利用围绕平直空间展开的式 (16) 给出的费曼规则，计算确定螺旋度 h 入射平面波的振幅 $\mathcal{A}_{\mu\nu}$ 。除引力子散射质量单极子 m 的项外，散射振幅中还存在来自式 (16) 接触项的贡献，例如，电勒夫算符贡献了一项形式为

$$\mathcal{A}_{\mu\nu} \supset k^E_{\ell=2}\mathcal{R}^5\omega^4\varepsilon^h_{\mu\nu}(k), \tag{30}$$

since each insertion of $E_{\mu\nu}$ acting on an asymptotic state brings down a factor of ω^2 . The short-distance part of the amplitude to scatter into a final state plane wave of helicity h' is therefore proportional to $\omega^4\varepsilon^{-h'}_{\mu\nu}(k')\,\varepsilon^{h\mu\nu}(k)$.

因为作用在渐近态上的每一次 $E_{\mu\nu}$ 插入都会带来一个因子 ω^2 。因此，散射到螺旋度为 h' 的末态平面波的振幅的短距离部分正比于 $\omega^4\varepsilon^{-h'}_{\mu\nu}(k')\,\varepsilon^{h\mu\nu}(k)$ 。

In general, the Love numbers are dependent on the form of the equation of state of the compact star. They were studied first in Refs. [38] for the case of neutron stars, by analyzing the perturbations to a background Oppenheimer-Volkoff model of relativistic stellar structure. See also [69,70]. For Schwarzschild black holes, the situation is somewhat trickier, for two reasons. (i). Technically $\mathcal{R}^5/G_N \propto G_N^4 m^5$, so that to match the EFT one must in principle subtract contributions to the amplitude that involve multiple (up to five) insertions of the mass m . (ii). Due to the presence of the event horizon, the low-frequency linear response of a black hole is necessarily absorptive. Because the local in-time point particle Lagrangian Eq. (4) cannot describe dissipative finite size effects, additional degrees of freedom need to be added to the worldline theory. Therefore, we postpone the topic of black hole response until the last part of this chapter; see section ”Event Horizon Dynamics in

一般而言，勒夫数依赖于致密星物态方程的形式。勒夫数最早在文献 [38] 中被研究，针对中子星情形，该研究通过对相对论恒星结构的奥本海默-沃尔科夫背景模型做扰动分析完成，另见文献 [69,70]。对于施瓦西黑洞，情况因两个原因变得有些复杂:(i). 从技术层面讲 $\mathcal{R}^5/G_N \propto G_N^4 m^5$ ，因此为了匹配有效场论，原则上必须对振幅中包含多次 (最多五次) 质量 m 插入的贡献做减除。(ii). 由于事件视界的存在，黑洞的低频线性响应必然存在吸收。因为式 (4) 的局域点粒子拉格朗日量无法描述耗散性有限尺寸效应，需要往世界线理论中添加额外的自由度。因此我们将黑洞响应这一主题推迟到本章最后一部分讨论，见章节“事件视界动力学

Regardless of the specific numerical value of $k_{\ell=2}^{E,B}$, the observation that $\ell = 2$ tidal deformability scales like $\mathcal{R}^5$ implies, by dimensional analysis, that tidal corrections in a compact binary in a non-relativistic orbital of radius $r \gg r_s$ do not enter until at least the relative order

无论 $k_{\ell=2}^{E,B}$ 的具体数值是多少，观测表明 $\ell = 2$ 潮汐形变率正比于 $\mathcal{R}^5$ ，通过量纲分析可得: 处于半径为 $r \gg r_s$ 的非相对论轨道上的致密双星，潮汐修正至少直到相对阶才会出现

$$(\mathcal{R}/r)^5 \sim (\mathcal{R}/r_s)^5 \times (G_N m/r)^5 \sim (\mathcal{R}/r_s)^5 v^{10}, \tag{31}$$

which is formally a 5PN effect, although possibly enhanced [38] for compact objects such as neutron stars with $\mathcal{R}/G_N m \sim \mathcal{O}(10)$. We have established, using EFT reasoning (symmetries, power counting), the ”Effacement Principle” [12] that non-dissipative finite size corrections cannot appear until 5PN order in the nonrelativistic limit. Thus in order to learn about the internal structure of compact objects during the adiabatic inspiral phase, one needs waveform templates that take into account the dynamics of point particles to 5PN beyond the quadrupole radiation formula. We turn to a review of such corrections next.

形式上这是一个 5PN(第 5 后牛顿) 效应，尽管对于存在 $\mathcal{R}/G_N m \sim \mathcal{O}(10)$ 的中子星这类致密天体，该效应可能会被增强 [38]。我们通过有效场论论证 (对称性、幂次计数) 得到了“消隐原理”[12]: 在非相对论极限下，非耗散的有限尺寸修正直到 5PN 阶才会出现。因此，若要在绝热旋进阶段研究致密天体的内部结构，就需要所用的波形模板考虑超出四极辐射公式、到 5PN 阶的点粒子动力学。我们接下来就回顾这类修正。

Perturbative Binary Dynamics

微扰双星动力学

Setup

准备工作

We now consider a binary merger of compact objects, restricting ourselves to the regime in which the typical orbital distance r is large, $r \gg \mathcal{R} \gtrsim r_s$. In this phase, the system is well described by Einstein gravity coupled to the point particle action S_{pp} , truncated at some fixed order in the derivative expansion. The goal is to predict the classical gravitational waveform $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ measured by detectors at $\mathcal{I}^+$, as well as the flux of energy, momentum, and angular momentum as a perturbative expansion in the small quantities $\mathcal{R}/r$ and r_s/r .

我们现在研究致密天体并合, 仅讨论典型轨道距离 r 很大 $r \gg \mathcal{R} \gtrsim r_s$ 的参数区间。在这个阶段, 该系统可以很好地用耦合点粒子作用量 S_{pp} 的爱因斯坦引力描述，并在导数展开中截断到某固定阶。我们的目标是, 对小量 $\mathcal{R}/r$ 和 r_s/r 做微扰展开, 预言探测器在 $\mathcal{I}^+$ 处测得的经典引力波形 $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ ，以及能量、动量和角动量的流量。

Even though we are mainly interested in solving a classical problem, it turns out to be convenient to set it up as the $\hbar \to 0$ limit of a quantum field theory calculation. The graviton $h_{\mu\nu}$ is treated as a propagating quantum field, but the binary constituents are classical worldline sources with zero quantum fluctuations. In this picture, the waveform then corresponds to an expectation value

尽管我们主要关注求解经典问题，但将该问题构造为量子场论计算的 $\hbar \to 0$ 极限会更为方便。引力子 $h_{\mu\nu}$ 被处理为传播的量子场，但双星的两个组成部分是不存在量子涨落的经典世界线源。在这个框架下，波形对应一个期望值

$$\langle \text{in} \left| h_{\mu\nu}(x) \right| \text{in} \rangle,$$

evaluated in the initial vacuum state of the radiation field and of the binary constituents. As was first emphasized in Ref. [46], the appropriate path integral formalism for computing this expectation value, one in which we hold the initial state fixed but sum over the final states of the radiation field, is the "Schwinger-Keldysh" [73, 104] closed time path (CTP) or "in-in" version of the functional integral. (See [19] for a review of the in-in formalism for quantum mechanics and field theory.) This is analogous to the situation in cosmology, where late time correlations are measured in a given initial state [116].

该期望值在辐射场与双星组分的初态真空下计算。正如文献 [46] 首先强调的，计算这个期望值的合适路径积分形式是“施温格-凯尔迪什”[73, 104] 闭时路径 (CTP)，或称“in-in”版本的泛函积分: 它固定初态，对辐射场的末态求和。(关于量子力学和量子场论的 in-in 形式，综述可见 [19]。) 这与宇宙学中的情况类似，在宇宙学中，晚期关联函数是在给定初态下测量得到的 [116]。

(As an aside, it is also possible to formulate the radiation problem as an S -matrix calculation [79], in which both the graviton and the massive particles are dynamical, and one uses time-ordered propagators in the Feynman rules. As in the worldline EFT, such an approach is restricted to observables that are calculable in perturbation theory, i.e., to times well before the coalescence of the binary into (presumably) a final state black hole. The IR safe observables are not the individual S -matrix elements but rather semi-inclusive averages that sum over all the unobserved final asymptotic states of the system. Evaluating such sums via unitarity cuts is equivalent to using in-in Feynman rules, where the cuts correspond to Wightman propagator exchange between sources on opposite sides of the closed time path; see Eq. (44).)

(插叙一句，辐射问题也可以构造为 S 矩阵计算 [79]: 在这种方法中引力子和大质量粒子都是动力学的，费曼规则中使用时序传播子。和世界线有效场论一样，这种方法仅适用于可微扰计算的可观测量，也就是双星并合为 (推测是) 末态黑洞之前很久的时间。红外安全可观测量不是单独的 S 矩阵元，而是半系综平均，对系统所有未被观测的末态渐近态求和。通过幺正割计算这类求和等价于使用 in-in 费曼规则，其中的割对应闭时路径两侧源之间交换的怀特曼传播子；见式 (44)。)

To calculate observables, we introduce a generating function, the in-in effective action $\Gamma[x_A, \bar{g}; \tilde{x}_A \tilde{g}]$, defined by the path integral expression:

为了计算可观测量，我们引入一个生成函数，即 in-in 有效作用量 $\Gamma[x_A, \bar{g}; \tilde{x}_A \tilde{g}]$ ，它由以下路径积分表达式定义:

$$e^{i\Gamma[x_A,\bar{g};\tilde{x}_A\bar{g}]} = \int \mathcal{D}h_{\mu\nu}(x)\,\mathcal{D}\widetilde{h}_{\mu\nu}(x)\,e^{iS[\bar{g},h,x_A]-iS[\bar{g},\widetilde{h},\tilde{x}_A]}. \tag{32}$$

Here, the classical action is a functional $S[\bar{g},h,x_A] = S_{EH}[\bar{g}+h] + S_{pp}[\bar{g}+h,x_A] + S_{GF}[\bar{g},h]$ of the integration variable $h_{\mu\nu}$, as well as the worldlines $x^{\mu}_{A=1,2}(\tau)$, which are held fixed to arbitrary values in the computation of the integral. The action also depends on a c -number background gravitational field $\bar{g}_{\mu\nu}$ which is also held fixed in the course of evaluating Eq. (32).

在此处，经典作用量是积分变量 $h_{\mu\nu}$ 以及世界线 $x^{\mu}_{A=1,2}(\tau)$ 的泛函 $S[\bar{g},h,x_A] = S_{EH}[\bar{g}+h] + S_{pp}[\bar{g}+h,x_A] + S_{GF}[\bar{g},h]$，在计算积分时，世界线被固定为任意值。该作用量还依赖于 c 数背景引力场 $\bar{g}_{\mu\nu}$，在计算式 (32) 的过程中，该引力场也保持固定。

It is convenient to employ the background field method [3, 31] , in which the gauge fixing term $S_{GH}[\bar{g},h]$ is chosen to preserve diffeomorphisms acting on $\bar{g}_{\mu\nu}$. For instance, the choice $S_{GF} = m_{\rm Pl}^{d-2}\int d^dx\sqrt{\bar{g}}\bar{g}^{\mu\nu}\Gamma_\mu\Gamma_\upsilon$, with $\Gamma_\mu = \overline{\nabla}_\rho h^\rho{}_\mu - \frac{1}{2}\overline{\nabla}_\mu h^\rho{}_\rho$ is background diffeomorphism invariant and generates graviton propagators whose Lorentz tensor structure in d spacetime dimensions is the standard one:

采用背景场方法 [3, 31] 会更为方便，该方法中选择规范固定项 $S_{GH}[\bar{g},h]$ 来保持作用在 $\bar{g}_{\mu\nu}$ 上的微分同胚不变性。例如，满足 $\Gamma_\mu = \overline{\nabla}_\rho h^\rho{}_\mu - \frac{1}{2}\overline{\nabla}_\mu h^\rho{}_\rho$ 的选择 $S_{GF} = m_{\rm Pl}^{d-2}\int d^dx\sqrt{\bar{g}}\bar{g}^{\mu\nu}\Gamma_\mu\Gamma_\upsilon$ 具有背景微分同胚不变性，由此得到的引力子传播子在 d 维时空下的洛伦兹张量结构是标准形式:

$$P_{\mu\upsilon;\rho\sigma} = \frac{1}{2}\left[\eta_{\mu\rho}\eta_{\upsilon\sigma} + \eta_{\mu\sigma}\eta_{\upsilon\rho} - \frac{2}{d-2}\eta_{\mu\upsilon}\eta_{\rho\sigma}\right]. \tag{33}$$

In the gauge specified by S_{GF} , it is also in principle necessary to introduce Faddeev-Popov ghost fields to ensure gauge invariance of the effective action. However, in the $\hbar \to 0$ limit which is the main focus of this review, ghost loop contributions to the path integral are subleading and may be ignored.

在 S_{GF} 指定的规范下，原则上也需要引入法捷耶夫-波波夫鬼场来保证有效作用量的规范不变性。但在本综述重点关注的 $\hbar \to 0$ 极限下，鬼圈对路径积分的贡献是次阶的，可以忽略。

Implicit in the integration measure of Eq. (32) is a choice of boundary conditions, corresponding to the vacuum in the far past for both $h_{\mu\nu}, \widetilde{h}_{\mu\nu}$. In the far future, the boundary condition is that $h_{\mu\nu} = \widetilde{h}_{\mu\nu}$, so roughly speaking, we may think of the arrow of time as starting at $t = -\infty$, going forward in time to $t = +\infty = \tilde{t}$ and then back to $\tilde{t} = -\infty$. From this point of view, the path integrals is over fields that propagate on the closed time contour from $-\infty$ back to ∞ in the forward sense, with $\widetilde{h}_{\mu\nu}$ field insertions occurring at times later than those of $h_{\mu\nu}$, at a time $t < \tilde{t}$.

式 (32) 的积分测度隐含了一组边界条件选择，对应 $h_{\mu\nu}, \widetilde{h}_{\mu\nu}$ 两者在远过去的真空。远未来的边界条件为 $h_{\mu\nu} = \widetilde{h}_{\mu\nu}$，因此粗略来说，我们可以认为时间箭头起始于 $t = -\infty$，沿时间正向行进到 $t = +\infty = \tilde{t}$，再返回 $\tilde{t} = -\infty$。从这个角度看，路径积分对沿闭合时间 contour 从 $-\infty$ 正向回到 ∞ 传播的场积分，场插入 $\widetilde{h}_{\mu\nu}$ 发生在比 $h_{\mu\nu}$ 更晚的时刻，即时刻 $t < \tilde{t}$。

The path integral Eq. (32) computes expectation values of operators which are time-ordered (τ -ordered) with respect to the closed time contour, e.g., at zero external field, an insertion of fields $h, \widetilde{h}$,

路径积分式 (32) 计算的是相对于闭合时间轮廓按时间排序 (τ 排序) 的算符的期望值，例如，在外场为零时，插入场 $h, \widetilde{h}$,

$$\mathcal{T}\left[\widetilde{h}(x_1)\cdots\widetilde{h}(x_k)\,h(x_{k+1})\cdots h(x_n)\right]$$

corresponds in canonical quantization to the operator product

在正则量子化中对应算符乘积

$$T^*\left[\widehat{h}(x_1)\cdots\widehat{h}(x_k)\right]\cdot T\left[\widehat{h}(x_{k+1})\cdots\widehat{h}(x_n)\right],$$

so that operator products are time-ordered (Feynman) or anti-time-ordered (Dyson) depending on which branch of the closed time path they lie on.

因此算符乘积是时间排序 (费曼) 还是反时间排序 (戴森)，取决于它们位于闭合时间路径的哪一分支。

In addition to the doubling of the integration variables relative to the standard time-ordered path integral for $|\text{in}\rangle \rightarrow|$ out $\rangle$ transition matrix elements, we also need to double the external classical sources in Eq. (32). By construction, the integral is normalized as $\Gamma[x_A, \bar{g}, x_A \bar{g}] = 0$, so to get any use out of the effective action, we need to first differentiate it with respect to ${x_A}^{\mu}$ or $\bar{g}_{\mu\nu}$ before setting $\tilde{\bar{g}}_{\mu\nu} = \bar{g}_{\mu\nu}$ or $\tilde{x}_A = x_A$.

相对于标准时间排序路径积分用于计算 $|\text{in}\rangle \rightarrow|$ 出 $\rangle$ 跃迁矩阵元，除了需要加倍积分变量，我们还需要对式 (32) 中的经典外源加倍。根据构造，积分按 $\Gamma[x_A, \bar{g}, x_A \bar{g}] = 0$ 归一化，因此要有效利用有效作用量，我们需要先对 ${x_A}^{\mu}$ 或 $\bar{g}_{\mu\nu}$ 求导，再令 $\tilde{\bar{g}}_{\mu\nu} = \bar{g}_{\mu\nu}$ 或 $\tilde{x}_A = x_A$ 取零。

In fact, all the relevant observables for radiation from the binary are obtained by functional differentiation of the in-in effective action, as we now explain. First, we solve the equations of motion for the worldlines x_A^{μ} , defined as the extrema of the in-in action

事实上，双星辐射的所有相关可观测量都可以通过对 in-in 有效作用量做泛函微分得到，我们现在对此进行说明。首先，我们求解世界线 x_A^{μ} 的运动方程，这些世界线定义为 in-in 作用量的极值点

$$\frac{\delta}{\delta x_A^{\mu}}\Gamma[x_A, \bar{g}, \tilde{x}_A \tilde{\bar{g}}]\Big|_{x_A=\tilde{x}_A;\bar{g},=\tilde{\bar{g}}=\eta} \equiv 0, \tag{34}$$

subject to a suitable set of initial data (particle positions and momenta). Because the in-in action is formally the result of integrating out gravitons that interact with the worldline and with themselves, the equations of motion are (at least at sufficiently high order in powers of G_N) in general nonlocal in time (see section ”Radiation Reaction in ($ELR%). They include both conservative and radiative corrections to the gravitational two-body dynamics. The solution of these equations of motion is then re-inserted into $\Gamma[x_A, \bar{g}, \tilde{x}_A, \tilde{\bar{g}}]$, yielding an ”on-shell” effective action for the background metric $\bar{g}_{\mu v}$.

受制于一组合适的初始数据 (粒子位置与动量)。由于入-作用量形式上是积分掉与世界线及自身相互作用的引力子后的结果，运动方程通常是时间非定域的 (至少在 G_N 幂次足够高的阶是如此)(参见“($ELR%) 中的辐射反作用”一节)。它们包含引力两体动力学的保守修正与辐射修正。再将这些运动方程的解代回 $\Gamma[x_A, \bar{g}, \tilde{x}_A, \tilde{\bar{g}}]$ ，就得到背景度规 $\bar{g}_{\mu\upsilon}$ 的“在壳”有效作用量。

Next, we vary this on-shell action with respect to $\bar{g}_{\mu\upsilon}$ to obtain the total energy-momentum pseudotensor $\tau_{\mu\upsilon}(x)$ of the binary system:

接下来，我们对 $\bar{g}_{\mu\upsilon}$ 变分这个在壳作用量，得到双星系统的总能动赝张量 $\tau_{\mu\upsilon}(x)$:

$$\tau_{\mu\upsilon} = \frac{2}{\sqrt{\bar{g}}} \frac{\delta}{\delta \bar{g}^{\mu\upsilon}} \Gamma[x_A, \bar{g}; \tilde{x}_A \tilde{\bar{g}}] \Big|_{x_A = \tilde{x}_A; \bar{g} = \tilde{\bar{g}} = \eta.} \tag{35}$$

Because of the choice of background field gauge, this pseudotensor is conserved on-shell, $\partial_\upsilon \tau^{\mu\upsilon} = 0$, but dependent on the gauge fixing term $S_{GF}[\bar{g}, h]$.

由于选取了背景场规范，该赝张量在壳守恒，$\partial_\upsilon \tau^{\mu\upsilon} = 0$ ，但依赖于规范固定项 $S_{GF}[\bar{g}, h]$ 。

Even though it depends on the choice of gauge, $\tau^{\mu\upsilon}$ has physical content, as the quantum mechanical amplitude for the binary system to emit an on-shell graviton of definite momentum k^μ out to $\mathcal{I}^+$:

尽管它依赖规范选取，$\tau^{\mu\upsilon}$ 仍具有物理意义，对应双星系统向外辐射一个确定动量的在壳引力子 k^μ 到 $\mathcal{I}^+$ 的量子力学振幅:

$$\mathcal{A}(k) = \varepsilon_{\mu\upsilon}(k) \mathcal{A}^{\mu\upsilon}(k) = -\frac{1}{2m_{Pl}} \int d^4 x e^{ik\cdot x} \varepsilon_{\mu\upsilon}(k) \tau^{\mu\upsilon}(x). \tag{36}$$

In turn, this on-shell amplitude has a simple relation to the waveform, once a gauge for the background field $\bar{h}_{\mu\nu} \equiv \bar{g}_{\mu\nu} - \eta_{\mu\nu}$ has been chosen. For instance, in deDonder gauge, the waveform at $|\mathbf{x}| \to \infty$ and fixed retarded time $u = t - |\mathbf{x}|$ is (setting $d = 4$) [115]

一旦选定背景场 $\bar{h}_{\mu\nu} \equiv \bar{g}_{\mu\nu} - \eta_{\mu\nu}$ 的规范，该在壳振幅与波形就存在简单关系。例如，在德东规范下，$|\mathbf{x}| \to \infty$ 处固定推迟时间 $u = t - |\mathbf{x}|$ 的波形为 (令 $d = 4$)[115]

$$\langle \text{in} | h_{\mu\nu}(x) | \text{in} \rangle \big|_{\mathcal{I}^+} = \frac{4G_N}{|\mathbf{x}|} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega u} \left[\mathcal{A}^{\mu\nu}(k) - \frac{1}{2} \eta^{\mu\nu} \mathcal{A}^{\rho}{}_{\rho}(k) \right], \tag{37}$$

where the on-shell momentum is $k^\mu = \omega(1, \mathbf{x}/r)$. Finally, the pseudotensor can also be used to calculate the flux of energy, momentum, and angular momentum radiated to $\mathcal{I}^+$, as well as the conserved ADM charges $P^\mu, J^{\mu\upsilon}$ of the system at spatial infinity i^0 , e.g.,

其中在壳动量为 $k^\mu = \omega(1, \mathbf{x}/r)$ 。最后，该赝张量还可用于计算辐射到 $\mathcal{I}^+$ 的能量、动量和角动量流，以及空间无穷远 i^0 处系统的守恒 ADM 荷 $P^\mu, J^{\mu\upsilon}$ ，例如，

$$\Delta P^\mu = \int \frac{d^d k}{(2\pi)^d} (2\pi) \delta(k^2) \theta(k^0) k^\mu \left| \mathcal{A}(k)_{\rho\sigma} \right|^2 \tag{38}$$

is the energy-momentum deposited in a detector placed at $\mathscr{I}^+$, summed over polarizations.

是放置在 $\mathscr{I}^+$ 处的探测器中沉积的能量动量，已对偏振求和。

In addition to being a useful generating function for observables, $\Gamma[x_A, \bar{g}; \tilde{x}_A, \tilde{\bar{g}}]$ also admits a convenient perturbative expansion in powers of G_N , in terms of Feynman diagrams. The Feynman rules for the graviton propagators and self-interaction vertices of the theory are derived in exactly the same way as in Yang-Mills theory (see, e.g., [35] for a review of the Feynman rules for gravity). There are also vertices generated by the coupling of gravitons to the classical worldlines in S_{pp} , for example, the term $-m\int d\tau$ in the point particle action generates a vertex with a single off-shell graviton of momentum k :

$\Gamma[x_A, \bar{g}; \tilde{x}_A, \tilde{\bar{g}}]$ 除了是可观测量的有用生成泛函外，还可以很方便地按 G_N 的幂次做微扰展开，用费曼图表示。该理论中引力子传播子和自相互作用顶点的费曼规则推导方式与杨-米尔斯理论完全相同 (例如参见 [35]，其中有对引力费曼规则的综述)。还有由 S_{pp} 中引力子与经典世界线耦合产生的顶点，例如点粒子作用量中的项 $-m\int d\tau$ 会生成一个顶点，对应一个动量为 k 的离壳单引力子:

$$\overset{k\to v^{\mu\nu}}{\sim} -\frac{im}{2m_{\rm Pl}^{d-2}}\int d\tau e^{ik\cdot x(\tau)}u^{\mu}(\tau)\,u^{\nu}(\tau)\,. \tag{39}$$

To calculate the functional integral, we also have keep track of field insertions on either side of the closed time contour, which requires a doubling of the fields in the theory, as mentioned above. Insertions of $h_{\mu\nu}$ and $\widetilde{h}_{\mu\nu}$ are time ordered with respect to an integration contour that starts at $t=-\infty$ and passes through $t=+\infty$ before bending back and ending at $\tilde{t}=-\infty$, as outlined above. Therefore the free field two-point functions are parsed as follows [18, 19] :

为了计算泛函积分，我们还必须追踪闭合时间轮廓两侧的场插入，这就如前文所述需要对理论中的场加倍。$h_{\mu\nu}$ 和 $\widetilde{h}_{\mu\nu}$ 的插入相对于积分轮廓是时序的，该轮廓起始于 $t=-\infty$ ，经过 $t=+\infty$ 后折回，终止于 $\tilde{t}=-\infty$ ，我们已经概述过。因此自由场两点函数可分解如下 [18, 19] :

$$\langle h_{\mu\upsilon}(x)\,h_{\rho\sigma}(y)\rangle = \left\langle 0\left|T\left[\hat{h}_{\mu\upsilon}(x)\,\hat{h}_{\rho\sigma}(y)\right]\right|0\right\rangle = P_{\mu\upsilon;\rho\sigma}D_F(x-y)\,, \tag{40}$$

$$\left\langle \widetilde{h}_{\mu\nu}(x)\,h_{\rho\sigma}(y)\right\rangle = \left\langle 0\left|\hat{h}_{\mu\nu}(x)\,\hat{h}_{\rho\sigma}(y)\right|0\right\rangle = P_{\mu\nu;\rho\sigma}W(x-y)\,, \tag{41}$$

$$\left\langle \widetilde{h}_{\mu\nu}(x)\,h_{\rho\sigma}(y)\right\rangle = \left\langle 0\left|T^*\left[\hat{h}_{\mu\nu}(x)\,\hat{h}_{\rho\sigma}(y)\right]\right|0\right\rangle = P_{\mu\nu;\rho\sigma}D_D(x-y)\,, \tag{42}$$

where

其中

$$D_F(x-y) = \int \frac{d^dk}{(2\pi)^d}e^{-ik\cdot(x-y)}\frac{i}{k^2+i0^+}, \tag{43}$$

is the propagator defined by the usual (time ordered) Feynman- $i\varepsilon$ contour prescription, $D_D(x-y) = [D_F(x-y)]^*$ is the Dyson anti-time-ordered propagator, and

是由常规 (时序) 费曼- $i\varepsilon$ 围道规定定义的传播子，$D_D(x-y) = [D_F(x-y)]^*$ 是戴森反时序传播子，且

$$W(x-y) = \langle 0|\phi(x)\phi(y)|0\rangle = \int \frac{d^d k}{(2\pi)^d}(2\pi)\delta\left(k^2\right)\theta\left(k^0\right)e^{-ik\cdot(x-y)} \tag{44}$$

is the Wightman function of a free massless field, which is evidently Green's function that propagates positive energy on-shell signals forward in time.

是自由无质量场的怀特曼函数，显然它就是在时间上向前传播正能在壳信号的格林函数。

The Feynman diagrams that contribute to the effective action are the ones that have internal $h_{\mu\nu}, \widetilde{h}_{\mu\nu}$ graviton lines coupled to classical worldline sources, as in Eq. (39), as well as diagrams with external (background gravitons) $\bar{h}_{\mu\nu}, \widetilde{\bar{h}}_{\mu\nu}$, as depicted in Fig. 2. In the $\hbar \to 0$ limit, we can drop graphs that have internal graviton loops or more than one insertion of $\bar{h}, \widetilde{\bar{h}}$.

对有效作用量有贡献的费曼图是带有内部 $h_{\mu\nu}, \widetilde{h}_{\mu\nu}$ 引力子线耦合到经典世界线源的图，如式 (39) 所示，也包括带有外 (背景引力子) $\bar{h}_{\mu\nu}, \widetilde{\bar{h}}_{\mu\nu}$ 的图，如图 2 所示。在 $\hbar \to 0$ 极限下，我们可以忽略带有内部引力子圈或多于一次 $\bar{h}, \widetilde{\bar{h}}$ 插入的图。

The framework outlined so far is well suited to calculate ”post-Minkowskian” (PM) corrections to two-body dynamics, in which the compact objects are fully relativistic, as a formal expansion in powers of G_N . Because weakly coupled relativistic particles do not form bound states, PM calculations are typically relevant for scattering processes at CM energies E_{CM} larger than the ADM masses of the compact objects and large impact parameter b such that the expansion parameter $G_N E_{\text{CM}}/b \ll 1$ is suitably small. In this limit the Feynman rules of the EFT provide a systematic simultaneous expansion in powers of $G_N E/b \ll 1$ (PM effects), $\hbar/L \ll 1$ (quantum corrections), where $L \sim Eb$ is the orbital angular momentum scale of the binary, and $(\mathcal{R}/b) \ll 1$ if one includes the finite size vertices in Eq. (4). The only physical scale appearing in any Feynman integral over internal graviton momenta is the separation b , so the effective theory has manifest powers counting in $G_N E/b \ll 1$, with $\hbar/L \ll 1$ serving to count the number of internal graviton loops in a given graph. Such PM scattering calculations have been the focus of intense study in recent years. For a recent review of recent developments and a more complete guide to the literature, see [14].

到目前为止概述的框架非常适合计算两体动力学的“后闵可夫斯基”(PM) 修正,其中致密天体是完全相对论性的,该修正为按 G_N 幂次的形式展开。由于弱耦合相对论粒子不会形成束缚态,PM 计算通常适用于质心系能量 E_{CM} 大于致密天体 ADM 质量、碰撞参数 b 足够大使得展开参数 $G_N E_{\text{CM}}/b \ll 1$ 足够小的散射过程。在此极限下，有效场论 (EFT) 的费曼规则给出了按 $G_N E/b \ll 1$ (PM 效应)、$\hbar/L \ll 1$ (量子修正) 幂次的系统性同时展开，其中 $L \sim Eb$ 是双星的轨道角动量标度，如果包含式 (4) 中的有限大小顶点则还有 $(\mathcal{R}/b) \ll 1$ 。对内部引力子动量的任意费曼积分中仅有的物理标度是间距 b ，因此有效理论在 $G_N E/b \ll 1$ 上有明确的幂次计数，而 $\hbar/L \ll 1$ 用于计数给定图中内部引力子圈的数量。这类 PM 散射计算近年来一直是研究热点。关于近期进展和更全面的文献指南，可见综述文献 [14]。

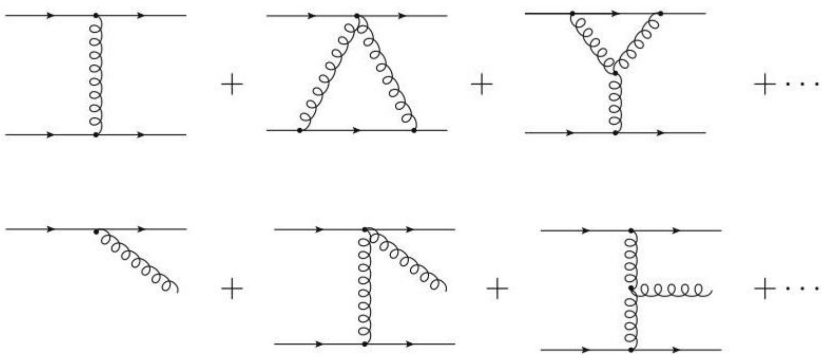


Fig. 2 Feynman diagram expansion of the in-in action. External lines correspond to insertions of the background field $\bar{h}_{\mu\nu}, \tilde{\bar{h}}_{\mu\nu}$. Each diagram shown stands for a sum of contributions from insertions on either side of the Schwinger-Keldysh closed time contour

图 2 进场作用量的费曼图展开。外线对应背景场 $\bar{h}_{\mu\nu}, \tilde{\bar{h}}_{\mu\nu}$ 的插入。图中每个图代表对在施温格-凯尔迪什闭合时间轮廓任意一侧插入的贡献求和

We will instead focus on the applications of Eq. (32) to PN calculations, where the expansion in powers of G_N and in powers of velocity become correlated. The formalism is not yet optimized for the computation of gravitational radiation from objects in bound non-relativistic orbits. Rather than integrating out the graviton in one fell swoop, in the non-relativistic case, it is useful instead to perform the inin path integral in successive stages, by factorizing the integration measure into modes with support near the orbital distance scale $\sim r$ and those localized around IR momentum scales $k^\mu \sim v/r$ corresponding to the frequency of the outgoing radiation. This Wilsonian, multi-step approach to bound state dynamics is described in detail starting in the next section.

我们将转而关注式 (32) 在 PN 计算中的应用，在 PN 计算中，按 G_N 的幂次展开和按速度的幂次展开是相互关联的。该形式体系目前尚未针对束缚非相对论轨道中天体的引力辐射计算优化。在非相对论情形下，不必一步到位积分 out 引力子，反而可以分阶段进行进场路径积分，将积分测度因子化为支撑在轨道距离标度 $\sim r$ 附近的模式，以及局域在对应出射辐射频率的红外动量标度 $k^\mu \sim v/r$ 附近的模式。从下一节开始，我们将详细描述这种用于束缚态动力学的威尔逊多步方法。

NRGR

非相对论广义相对论 (NRGR)

When $v \ll 1$, there exists a hierarchy of scales between orbital dynamics at distances $\sim r$ and radiation emission at wavelengths $\lambda \sim r/v \gg r$. While the Feynman diagrams described in the previous section scale as definite powers of $G_N M/r$ and of $\hbar/L$, they also contribute, after expanding in $v \ll 1$, at all orders in powers of velocity. Because in the PN expansion we are trying to determine the gravitational wave observables up to a fixed finite order in powers of v , it is desirable to have a perturbative scheme in which the Feynman rules are homogeneous in velocity, so that each diagram scales as a fixed power of v that can be determined from a simple set of power counting rules.

当 $v \ll 1$ 时，距离尺度为 $\sim r$ 的轨道动力学与波长尺度为 $\lambda \sim r/v \gg r$ 的辐射辐射之间存在标度层级。上一节描述的费曼图虽然按 $G_N M/r$ 和 $\hbar/L$ 的确定幂次标度，但在对 $v \ll 1$ 展开后，它们也会对速度的所有阶幂次产生贡献。由于在后牛顿 (PN) 展开中，我们需要将引力波观测量确定到 v 幂次的固定有限阶，因此希望存在这样一种微扰方案: 其中费曼规则对速度是齐次的，因此每个图都对应 v 的一个固定幂次，可以通过一组简单的幂次计数规则直接确定。

The lack of manifest low velocity scaling in the PM expansion can be traced to the presence of multiple scales in the momentum space Feynman integrals of the theory. When defined using dimensional regularization in $d = 4 - \varepsilon$ dimensions, momentum space integrals receive non-vanishing contributions from two types of kinematic regions [51]:

后闵可夫斯基 (PM) 展开中不存在明显的低速标度性，根源在于该理论动量空间费曼积分中存在多标度。当在 $d = 4 - \varepsilon$ 维中采用维数正规化定义时，动量空间积分会得到来自两类运动学区域的非零贡献 [51]:

- Potential: $\left(p^0, \mathbf{p}\right) \sim (v/r, 1/r)$,

- 势场区: $\left(p^0, \mathbf{p}\right) \sim (v/r, 1/r)$,

- Radiation: $\left(k^0, \mathbf{k}\right) \sim (v/r, v/r)$.

- 辐射区: $\left(k^0, \mathbf{k}\right) \sim (v/r, v/r)$.

The potential region corresponds to off-shell gravitons which are exchanged between the point sources. In position space, they generate the nearly instantaneous in time, long-range conservative gravitational forces responsible for binding the particles into closed orbits. On the other hand, radiation gravitons can go on-shell, propagating out to the detector, or remain off-shell, generating both ”dissipative” (time reversal odd) and ”conservative” (T -even) radiation reaction forces.

势场区对应在点源之间交换的离壳引力子。在位置空间中，它们产生几乎瞬时的长程保守引力，正是这种引力将粒子束缚为闭合轨道。另一方面，辐射引力子可以处于在壳态，向外传播到探测器，也可以保持离壳态，同时产生“耗散”(时间反演奇) 和“保守”(T -even) 辐射反作用力。

In dimensional regularization, a given Feynman integral can be ”threshold expanded” [9] around the various kinematic configurations of potential and radiation regions set by the external momenta (”method of regions”). The expanded Feynman integral is equal to a sum of simpler integrals, each characterized by a single physical scale. These simplified integrals can now be calculated for arbitrary (bound or unbound) non-relativistic trajectories $\mathbf{x}_{1,2}(t)$, as is necessary for the inspiral problem, and scale homogeneously as definite powers of the expansion parameter v .

在维数正规化中, 任意给定的费曼积分都可以在由外动量确定的势场区和辐射区的不同运动学构型附近做“阈值展开”[9]，即“区域方法”。展开后的费曼积分可写为多个更简单积分的和，每个简单积分仅由一个单一物理标度表征。这些简化积分现在可以对任意 (束缚或非束缚) 非相对论轨迹 $\mathbf{x}_{1,2}(t)$ 计算，满足旋近问题的要求，并且作为展开参数 v 的确定幂次齐次标度。

Rather than expanding out the PM momentum integrals in powers of v , we perform the expansion at the level of the Lagrangian, so as to obtain vertices with definite velocity scaling. By assumption, in the PN limit, there is a nearly inertial frame (e.g., the CM frame) where both sources are non-relativistic. Working in such a frame, we expand the worldline action explicitly in powers of velocity, e.g.,

我们没有将 PM 动量积分按 v 的幂次展开，而是在拉格朗日量层面完成展开，从而得到具有确定速度标度的顶点。根据假设，在 PN 极限下存在一个近似惯性系 (例如质心系)，其中两个源都是非相对论的。在这样的参考系中，我们将世界线作用量明确按速度幂次展开，例如:

$$
\begin{aligned}
-m\int d\tau &\supset -\frac{m}{2m_{\mathrm{Pl}}}\int d\tau h_{\mu\nu}u^{\mu}u^{\nu}+\cdots \\
&\approx -\frac{m}{2m_{\mathrm{Pl}}}\int dt\left[h_{00}+2h_{0i}\mathbf{v}^{i}+h_{ij}\mathbf{v}^{i}\mathbf{v}^{j}-\frac{1}{2}h_{00}\mathbf{v}^{2}\right]+\mathcal{O}\left(v^{3}\right).
\end{aligned} \tag{45}
$$

More crucially, we also perform at the level of the Lagrangian an explicit mode decomposition of the graviton into distinct potential and radiation fields, $\left(\int_{\mathbf{p}}\equiv\int d^{d-1}\mathbf{p}/(2\pi)^{d-1}\right)$

更关键的是，我们还在拉格朗日量层面将引力子显式模分解为不同的势场和辐射场，$\left(\int_{\mathbf{p}}\equiv\int d^{d-1}\mathbf{p}/(2\pi)^{d-1}\right)$

$$
h_{\mu\nu}(x)=\bar{h}_{\mu\nu}(x)+\int_{\mathbf{p}}e^{i\mathbf{p}\cdot\mathbf{x}}H_{\mu\nu;\mathbf{p}}\left(x^{0}\right). \tag{46}
$$

The radiation field $\bar{h}_{\mu\nu}$ has momentum scaling $k^{\mu}\sim(v/r,v/r)$, and thus spacetime derivatives ∂_{μ} acting on such fields are power counted as

辐射场 $\bar{h}_{\mu\nu}$ 的动量标度为 $k^{\mu}\sim(v/r,v/r)$ ，因此作用在这类场上的时空导数 ∂_{μ} 在拉格朗日量层面的幂次计数规则为

$$
\partial_{\rho}\bar{h}_{\mu\nu}\sim\frac{v}{r}\bar{h}_{\mu\nu} \tag{47}
$$

at the level of the Lagrangian. The potential graviton field $H_{\mu\nu;\mathbf{p}}\left(x^{0}\right)$ is defined in a mixed time/spatial momentum representation, such that the large momentum scale $\mathbf{p}\sim 1/r$ associated with virtual exchange between the particle sources is explicit. This way, spacetime derivatives acting on the re-phased potential mode $H_{\mu\nu;\mathbf{p}}\left(x^{0}\right)$ also scale as $\partial_{\mu}\sim v/r$, and therefore every derivative appearing in the Lagrangian of the theory is treated on the same footing.

势引力子场 $H_{\mu\nu;\mathbf{p}}\left(x^{0}\right)$ 定义在混合时间/空间动量表象中，因此与粒子源之间虚交换相关的大动量标度 $\mathbf{p}\sim 1/r$ 是显式的。通过这种方式，作用在重相位势模 $H_{\mu\nu;\mathbf{p}}\left(x^{0}\right)$ 上的时空导数也按 $\partial_{\mu}\sim v/r$ 标度，因此该理论拉格朗日量中出现的所有导数都可以按同一规则处理。

Inserting the mode expansion Eq. (46) into the action, we can read off the potential propagator from the $\mathcal{O}\left(H^{2}\right)$ part of the Einstein-Hilbert Lagrangian

将模展开式 (46) 代入作用量，我们可以从爱因斯坦-希尔伯特拉格朗日量的 $\mathcal{O}\left(H^{2}\right)$ 部分直接读出势传播子

$$\mathcal{L}_{H^2} = -\frac{1}{2}\int_{\mathbf{p}}\left[\mathbf{p}^2 H_{\mathbf{p}\mu\nu}H^{\mu\nu}_{-\mathbf{p}} - \frac{\mathbf{p}^2}{2}H_{\mathbf{p}}H_{-\mathbf{p}} - \partial_0 H_{\mathbf{p}\mu\nu}\partial_0 H^{\mu\nu}_{-\mathbf{p}} + \frac{1}{2}\partial_0 H_{\mathbf{p}}\partial_0 H_{-\mathbf{p}}\right].$$

(48)

The last two terms are $O\left(v^2\right)$ relative to the first two and are treated perturbatively, as insertions, in the path integral. They encode retardation effects due to the finite speed of light. On the other hand, the first two terms determine the propagator

> 后两项相对于前两项是 $O\left(v^2\right)$ 阶的，在路径积分中作为插入项按微扰处理。它们描述光速有限带来的推迟效应，而前两项则决定了传播子

$$\left\langle H_{\mathbf{p}\mu\nu}\left(x^0\right)H_{\mathbf{q}\rho\sigma}(0)\right\rangle = -(2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})\frac{i}{\mathbf{p}^2}P_{\mu\nu;\rho\sigma}\delta\left(x^0\right), \tag{49}$$

which is instantaneous in time. Note that because the potential modes do not go on-shell, $\mathbf{p}^2 \neq 0$, an $i\varepsilon$ contour prescription is not needed to define the free two-point function. Given that $x^\mu \sim v/r$ and $\mathbf{p} \sim 1/r$, this equation tells us that we should assign the scaling rule

> 其在时间上是瞬时的。注意由于势模不会在壳，$\mathbf{p}^2 \neq 0$ ，因此不需要 $i\varepsilon$ 围道规定来定义自由两点关联函数。已知 $x^\mu \sim v/r$ 和 $\mathbf{p} \sim 1/r$ ，该式表明我们应当赋予标度规则

$$H_{\mathbf{p}} \sim r^2\sqrt{v} \tag{50}$$

at the level of the EFT Lagrangian.

> 在有效场论拉格朗日量层面。

The scaling rule for radiation is more straightforward. The radiation graviton propagator scales as $1/k^2 \sim (r/v)^2$ in momentum space so that in position space $\langle h(x)\,h(0)\rangle \sim \int d^4k/k^2 \sim (v/r)^2$, and therefore we assign the rule

> 辐射的标度规则更直接。辐射引力子传播子在动量空间标度为 $1/k^2 \sim (r/v)^2$ ，因此位置空间中为 $\langle h(x)\,h(0)\rangle \sim \int d^4k/k^2 \sim (v/r)^2$ ，我们赋予规则

$$\bar{h}_{\mu v}(x) \sim v/r \tag{51}$$

to insertions of the radiation field.

> 给辐射场的插入项。

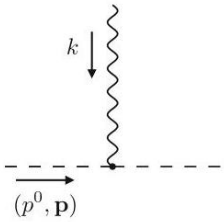
The final step in constructing an EFT with definite velocity scaling is to multipole expand the radiation field at the level of the action, either in its couplings to the particle sources or to the potentials [63]. To see why this is necessary, consider, e.g., the amplitude for a particle to absorb a single radiation graviton, given in Eq. (39). In the non-relativistic limit, the exponential phase $e^{ik\cdot x}$ in the amplitude contains a factor

构造具有确定速度标度的有效场论的最后一步，是在作用量层面将辐射场进行多极展开，无论是对其与粒子源的耦合还是与势的耦合 [63]。为说明这一操作的必要性，例如考虑式 (39) 给出的粒子吸收单个辐射引力子的振幅。在非相对论极限下，振幅中的指数相位 $e^{ik\cdot x}$ 包含一个因子

$$e^{-i\mathbf{k}\cdot\mathbf{x}} \sim e^{\mathcal{O}(v)} \tag{52}$$

that does not scale homogeneously in the expansion parameter, given that the particle orbits $\mathbf{x} \sim r$ emit radiation of typical momentum $\mathbf{k} \sim v/r$. Similarly, consider a generic Feynman diagram that includes absorption of radiation by an internal potential graviton, e.g.,

其在展开参数中不满足齐次标度，因为粒子轨道运动 $\mathbf{x} \sim r$ 辐射出典型动量为 $\mathbf{k} \sim v/r$ 的辐射。类似地，考虑包含内部势引力子吸收辐射的一般费曼图，例如



(53)

In this diagram, the outgoing potential graviton comes with a propagator $\propto 1/(\mathbf{p}+\mathbf{k})^2$, so given that $\mathbf{p} \sim 1/r, \mathbf{k} \sim v/r$, it gives contributions at all orders in powers of v .

在该图中，出射势引力子带有一个传播子 $\propto 1/(\mathbf{p}+\mathbf{k})^2$ ，因此已知 $\mathbf{p} \sim 1/r, \mathbf{k} \sim v/r$ ，它会贡献 v 各阶的项。

The remedy in either case is to substitute the multipole expanded radiation field

两种情况的解决方法都是代入多极展开后的辐射场

$$\bar{h}_{\mu\nu}(x) \mapsto \sum_{\ell=0}^{\infty} \frac{1}{\ell!} \mathbf{x}^{i_1} \cdots \mathbf{x}^{i_\ell} \partial_{i_1} \cdots \partial_{i_\ell} \bar{h}_{\mu\nu}(x^0, 0), \tag{54}$$

into the terms in the Lagrangian involving radiation couplings to either potentials or the worldlines (the radiation mode self-interactions do not get multipole expanded). Here, we have chosen coordinates such that $\mathbf{x} = 0$ lies somewhere near the center of mass of the binary system and $\mathbf{x} \sim r$ is a point somewhere inside the binary system. (The precise choice of center for the multipole decomposition is arbitrary and chosen for computational convenience. It has no effect on physical observables.) The ℓ -th order term in the multipole expansion is suppressed by v^ℓ relative to the monopole ($\ell = 0$) term, and in practical calculations at fixed PN accuracy, the expansion may be truncated at a finite value of ℓ .

到拉格朗日中涉及辐射与势或世界线耦合的项中 (辐射模自相互作用不需要多极展开)。此处我们选取坐标系使得 $\mathbf{x}=0$ 位于双星系统质心附近，而 $\mathbf{x}\sim r$ 是双星系统内部的某一点。(多极分解中心的具体选择是任意的，仅为计算方便，对物理可观测量没有影响。) 多极展开中 ℓ 阶项相对于单极 ($\ell=0$) 项被 v^ℓ 压低，在固定后牛顿精度的实际计算中，展开可以在 ℓ 的有限阶截断。

Given the decomposition of the graviton into potentials and radiation, together with the multipole expansion of radiation, we now have a set of rules that allow us to count powers of v either at the level of the action or inside correlation functions. These rules are summarized in the following table:

在将引力子分解为势模和辐射模、并对辐射做了多极展开之后，我们现在得到了一套规则，可以在作用量层面或关联函数内部对 v 的幂次计数。这些规则总结在下表中:

∂_μ	x^μ	$\mathbf{x}_a$	$\mathbf{p}$	k^μ	$H_{\mathbf{p}}$	$\hbar$	m/m_{Pl}
v/r	r/v	r	$1/r$	v/r	$r^2\sqrt{v}$	v/r	$\sqrt{Lv}$.

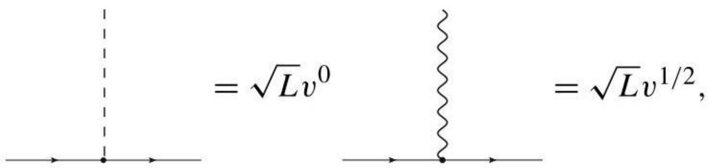
Using these rules, one finds by dimensional analysis that every Feynman diagram scales as a definite power of the angular momentum scale $L=mvr$ times a definite power of v . For example, a tree-level diagram (no internal graviton loops) gives a contribution to the effective action Eq. (32) of order $\sim L^{1-n/2}$, where n is the number of external radiation graviton lines. Each additional internal graviton loop costs a factor of $1/L$. The classical limit corresponds to orbital angular momenta $L\gg\hbar$, in which case one can simply ignore graphs with internal loops or more than one external leg.

利用这些规则，通过量纲分析可以发现，每个费曼图的标度对应角动量标度 $L=mvr$ 的一个确定幂次乘以 v 的一个确定幂次。例如，树图 (无内部引力子圈) 对有效作用量 (32) 式的贡献阶数为 $\sim L^{1-n/2}$ ，其中 n 是外部辐射引力子线的数量。每增加一个内部引力子圈，就会多出一个因子 $1/L$ 。经典极限对应轨道角动量 $L\gg\hbar$ ，此时可以直接忽略含内部圈或多于一条外腿的图。

For example, the leading order coupling of the worldline to potential (dashed) or radiation (wavy) gravitons scale as

例如，世界线对势引力子 (虚线) 或辐射引力子 (波浪线) 的领头阶耦合标度为

(55)

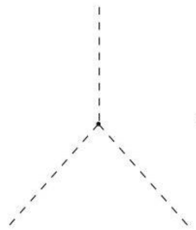


and therefore the leading order Newton potential exchange between two particles, involving two insertions of the vertex on the left, is of order $\sim Lv^0$ in the PN expansion. Similarly, the $H^3, H^2\bar{h}$ interaction vertices correspond to

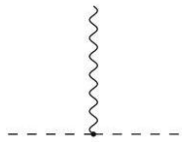
因此，涉及左侧顶点两次插入的两粒子间领头阶牛顿势交换，在 PN 展开中阶数为 $\sim Lv^0$ 。类似地，$H^3, H^2\bar{h}$ 相互作用顶点对应

(56)and so on.

依此类推。



$$= \frac{v^2}{\sqrt{L}}$$



$$= \frac{v^{5/2}}{\sqrt{L}},$$

We will refer to the theory with both potentials and radiation as NRGR [51] to emphasize the analogy to NRQCD [20], the EFT description of non-relativistic heavy quark bound states $Q\bar{Q}\left(M_Q \gg \Lambda_{QCD}\right)$, which employs a similar mode decomposition [84], and multipole expansion [63] of the gluon fields in full QCD. In practical calculations, NRGR is used to interpolate between the theory of relativistic particles in Eq. (3) in the UV and another EFT in the IR that results from integrating out the potential gravitons. This EFT streamlines the calculation of radiative corrections to bound state dynamics, as described in the next section.

我们将同时包含势场和辐射的这套理论称为 NRGR [51]，以强调它与非相对论重夸克束缚态的 EFT 描述 NRQCD [20] 的类比——$Q\bar{Q}\left(M_Q \gg \Lambda_{QCD}\right)$，后者在全 QCD 中对胶子场采用了类似的模式分解 [84] 和多极展开 [63]。在实际计算中，NRGR 用于连接紫外区 (3) 式的相对论粒子理论和红外区积分掉势引力子后得到的另一个 EFT。如下文所述，该 EFT 简化了束缚态动力学辐射修正的计算。

Radiative Corrections and ZSO IR

辐射修正与 ZSO 红外有效理论

Because the potential gravitons cannot go on-shell, it is possible to integrate them out to obtain a local EFT of self-interacting radiation gravitons coupled to the bound state. We refer to this EFT, valid at distances longer than the orbital scale r , by the acronym "Z'So IR", since it encodes the interactions of a Zoomed Out Single Object whose internal structure, i.e., the binary constituents, cannot be directly resolved by long wavelength radiation modes (the subscript "IR" is to distinguish this EFT from a similar theory of black hole horizon fluctuations in the UV that will be discussed in section "Event Horizon Dynamics in TSOUV").

由于势引力子无法在壳，我们可以将其积出，得到自相互作用辐射引力子耦合到束缚态的局域有效场论。这个适用于长于轨道尺度 r 距离的有效场论，我们缩写为“ZSO IR”，因为它描述的是一个缩并单物体的相互作用，其内部结构 (即双天体组分) 无法被长波长辐射模式直接分辨 (下标“IR”用于区分该有效场论与将在“TSOUV 事件视界动力学”章节讨论的紫外波段黑洞视界涨落的类似理论)。

Like the gapped point particle of section "The One-Body Sector," the composite object is defined in terms of a center-of-mass variable $x^\mu(\tau)$ and an orthonormal frame $e^a{}_\mu$ that accounts for the spatial orientation relative to asymptotic inertial observers. In addition, it contains an infinite tower of multipole moments $I_{a_1 \cdot a_\ell}$

, $J_{a_1 \cdot a_\ell}$ of parity $(-1)^{\ell}, (-1)^{\ell+1}$, respectively, coupled to the Weyl curvature of the radiation field. The form of the worldline action follows [52, 55, 95] from diffeomorphism invariance:

和“单体 sector”章节的有能隙点粒子类似，该复合物体由质心变量 $x^\mu(\tau)$ 和一个正交标架 $e^a{}_\mu$ 定义，正交标架描述其相对于渐近惯性观测者的空间取向。此外，它包含无穷多极矩序列 $I_{a_1 \cdot a_\ell}$ 、$J_{a_1 \cdot a_\ell}$ ，分别耦合到宇称 $(-1)^{\ell}, (-1)^{\ell+1}$ ，对应辐射场外尔曲率。世界线作用量的形式由微分同胚不变性给出 [52, 55, 95] :

$$S_{TT_{\rm IR}} = S_{EH} + \int d\tau L\left(X, x\left(\tau\right), \bar{g}\right) + \frac{1}{2}\int d\tau S_{ab}\left(\tau\right)\Omega^{ab} + \frac{1}{2}\int d\tau I_{ab}\left(\tau\right)E^{ab}\left(x\left(\tau\right)\right)$$

$$-\frac{2}{3}\int d\tau J_{ab}\left(\tau\right)B^{ab}\left(x\left(\tau\right)\right) + \frac{1}{6}\int d\tau I_{abc}\left(\tau\right)\nabla^c E^{ab}\left(x\left(\tau\right)\right) + \cdots. \tag{57}$$

We proceed to explain the meaning of the various terms in this equation.

接下来我们解释该方程中各项的含义。

First, the term

首先，该项

$$S_X = \int d\tau L\left(X, x\left(\tau\right), \bar{g}\right) \tag{58}$$

provides the dynamics of the internal degrees of freedom ” X ” of the composite system in the absence of radiation, $E_{ab} = B_{ab} = 0$. For gapped constituents in a non-relativistic bound state,” X ” stands in for the coordinates $\mathbf{x}_{1,2}$ and the spins $\mathbf{S}_{1,2}$, but we can consider a more general situation in which each particle carries additional low-lying modes (see section ”Event Horizon Dynamics in Tab UV”). The Lagrangian $L\left(X, x^\mu\left(\tau\right), \bar{g}_{\mu\nu}\right)$ determines the ADM mass of the composite object, obtained by expanding to linear order in $\bar{h}_{\mu\nu}$. Even though we have chosen to parameterize the action Eq. (57) in terms of the proper time along $x^\mu\left(\tau\right)$, the Hamiltonian for the system is non-zero

给出了无辐射时复合系统内自由度“X ”的动力学，$E_{ab} = B_{ab} = 0$ 。对于非相对论束缚态中的有能隙组分，“X ”代表坐标 $\mathbf{x}_{1,2}$ 和自旋 $\mathbf{S}_{1,2}$ ，但我们可以考虑更一般的情况: 每个粒子携带额外低能模式 (参见“Tab UV 事件视界动力学”章节)。拉格朗日量 $L\left(X, x^\mu\left(\tau\right), \bar{g}_{\mu\nu}\right)$ 决定了复合物体的 ADM 质量，该质量可通过展开到 $\bar{h}_{\mu\nu}$ 的一阶得到。尽管我们选择用沿 $x^\mu\left(\tau\right)$ 的固有时参数化 (57) 式的作用量，该系统的哈密顿量非零

$$H_X = \left[\frac{dX}{d\tau}\frac{\partial}{\partial \dot{X}}L\left(X, x\left(\tau\right), \bar{g}\right) - L\left(X, x\left(\tau\right), \bar{g}\right)\right], \tag{59}$$

since excitations localized on the composite worldline acts like an internal clock that spontaneously break the time reparameterization gauge symmetry of the underlying theory.

因为局域在复合世界线上的激发相当于一个内部时钟, 会自发破缺基础理论的时间重参数化规范对称性。

Ignoring spin for simplicity (and still keeping $E_{ab} = B_{ab} = 0$), the equations of motion for x^{μ} imply that $p^{\mu} = H_X(\tau)\, dx^{\mu}/d\tau$ follows a geodesic of $\bar{g}_{\mu\nu}$, while the Euler-Lagrange equations for X imply that H_X is a constant on-shell. By comparing the asymptotic (linearized) gravitational field of the composite particle, sourced by

为简化讨论忽略自旋 (但仍保留 $E_{ab} = B_{ab} = 0$)， x^{μ} 的运动方程表明 $p^{\mu} = H_X(\tau)\, dx^{\mu}/d\tau$ 沿 $\bar{g}_{\mu\nu}$ 的测地线运动，而 X 的欧拉-拉格朗日方程表明 H_X 在壳为常数。通过比较由下式源出的复合粒子渐近 (线性化) 引力场

$$T_X^{\mu\nu}(x) = \frac{2}{\sqrt{\bar{g}}}\frac{\delta}{\delta g^{\mu\nu}(x)} S_X = \int d\tau H_X \frac{\delta^4(x - x(\tau))}{\sqrt{\bar{g}}}\frac{dx^{\mu}}{d\tau}\frac{dx^{\nu}}{d\tau}, \tag{60}$$

with the full theory, Eq. (8), we can identify H_X , evaluated on-shell, with the ADM mass of the system in the CM frame, Eq. (9).

和完整理论的 (8) 式，我们可以将壳求值后的 H_X 识别为质心系下系统的 ADM 质量，即 (9) 式。

Thus, in flat space, the motion of the worldline is simply $x^{\mu}(\tau) = u^{\mu}\tau + x^{\mu}{}_{\rm CM}$, with $x^{\mu}_{\rm CM}$ and u^{μ} which are constants. Including also the spin [66], the term $-\int d\tau S_{ab}\Omega^{ab}$ (the angular velocity Ω_{ab} is defined in Eq. (2)) then gives the ADM angular momentum of the system as

因此，在平直空间中，世界线的运动简单为 $x^{\mu}(\tau) = u^{\mu}\tau + x^{\mu}{}_{\rm CM}$ ，其中 $x^{\mu}_{\rm CM}$ 和 u^{μ} 均为常数。纳入自旋后 [66]，项 $-\int d\tau S_{ab}\Omega^{ab}$ (角速度 Ω_{ab} 定义见 (2) 式) 给出系统的 ADM 角动量为

$$J^{\mu\nu} = x^{\mu}_{\rm CM} P^{\nu} - x^{\mu}_{\rm CM} P^{\nu} + S^{\mu\nu}, \tag{61}$$

$S^{\mu\nu} = e^{\mu}{}_a e^{\nu}{}_b S^{ab}$, which is also conserved by the equations of motion.

$S^{\mu\nu} = e^{\mu}{}_a e^{\nu}{}_b S^{ab}$ ，它也满足运动方程守恒。

One can think of the moments $S_{ab}, I_{ab}, J_{ab}, \ldots$ as (time-dependent) Wilson coefficients that encode the internal structure of the radiating system. They are, in general, functions of the internal variables X and are obtained by matching to the UV theory, as we will describe in more detail below. We have chosen to define these moments relative to the local frame e^a_{μ} and to classify them in terms of representations of a $SO(3) \subset SO(3,1)$ subgroup of the local Lorentz transformations that leaves $P^a = e^a{}_{\mu} P^{\mu}$ invariant (The ℓ -th order moments $I_{a_1 \cdots a_\ell}, J_{a_1 \cdots a_\ell}$ fit together into a representation $(\ell, 0) \oplus (0, \ell)$ os $SO(3,1)$.). Similarly the electric and magnetic curvatures $E_{ab} = e^{\mu}{}_a e^{\nu} E_{\mu\nu}, \nabla_a E_{bc} = e^{\mu}{}_a e^{\nu}{}_b e^{\rho}{}_c \nabla_{\mu} E_{\nu\rho}$, etc. are projections onto the rotating frame $e^a{}_{\mu}$.

我们可以将矩 $S_{ab}, I_{ab}, J_{ab}, \ldots$ 视为 (含时的) 威尔逊系数，它们编码辐射系统的内部结构。一般而言，它们是内部变量 X 的函数，通过匹配紫外理论得到，我们将在下文更详细说明。我们选择相对于局部坐标系 e^a_{μ} 定义这些矩，并按照使 $P^a = e^a{}_{\mu} P^{\mu}$ 不变的局部洛伦兹变换的 $SO(3) \subset SO(3,1)$ 子群的表示对它们分类 (ℓ 阶矩 $I_{a_1 \cdots a_\ell}, J_{a_1 \cdots a_\ell}$ 共同构成 $SO(3,1)$ 的表示 $(\ell, 0) \oplus (0, \ell)$)。类似地，电曲率和磁曲率 $E_{ab} = e^{\mu}{}_a e^{\nu} E_{\mu\nu}, \nabla_a E_{bc} = e^{\mu}{}_a e^{\nu}{}_b e^{\rho}{}_c \nabla_{\mu} E_{\nu\rho}$ 等也是投影到旋转坐标系 $e^a{}_{\mu}$ 得到的结果。

The full dynamics of the non-relativistic bound state consists of Eq. (57) coupled to the Einstein-Hilbert term for the radiation metric $g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu}$. It is worth noting that Eq. (57) is actually universal, in the sense that it gives a description of soft gravitational radiation from a completely generic self-gravitating object of finite spatial extent $\sim b$ and ADM energy $\sim E$. For such a system, we necessarily (Assuming that any physically reasonable system that is squeezed down to a size smaller than its Schwarzschild radius will inevitably collapse gravitationally into a black hole [90].) have $G_N E \lesssim b$ and multipole moments of generic size $\sim Eb^{\ell}$. Therefore the emission or absorption of multipole radiation with $\omega b \ll 1$ can be described systematically by this EFT.

非相对论束缚态的完整动力学由式 (57) 耦合辐射度规 $g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu}$ 的爱因斯坦-希尔伯特项描述。值得注意的是，式 (57) 实际上具有普适性: 它描述了任意有限空间范围 $\sim b$ 、ADM 能量为 $\sim E$ 的一般自引力物体的软引力辐射。对于这类系统，我们必然有 $G_N E \lesssim b$ ，且多极矩为一般大小 $\sim Eb^{\ell}$ (假设任何物理上合理的系统被压缩到小于史瓦西半径的尺寸后，都会不可避免地发生引力坍缩形成黑洞 [90])。因此，满足 $\omega b \ll 1$ 的多极辐射的发射或吸收都可以通过这个有效场论系统描述。

At the classical level, the EFT computes perturbative corrections as a double expansion in powers of $G_N E\omega \ll 1$, which controls effects due to graviton propagation in the curved spacetime sourced by the object and the multipole expansion parameter $\omega b \ll 1$. In principle, the EFT can also calculate quantum corrections due to graviton loops, which are suppressed by powers of $\hbar/Eb \ll 1$ and completely negligible for astrophysical applications. Because $G_N E\omega \ll 1$, the Feynman rules are those of flat space gravity, coupled to sources localized on a worldline. By power counting, one finds that in the classical limit $\hbar/Eb \ll 1$, only the diagrams with at most a single external graviton survive, of the form shown in Fig. 3. The resulting Feynman integrals are tractable by well-established techniques [117] and, at least at sufficiently small orders in $G_N E/b \ll 1$, are calculable analytically for arbitrary time-dependent source moments $I_{a_1\cdots a_\ell}, J_{a_1\cdots a_\ell}$.

经典层面上，有效场论将微扰修正计算为 $G_N E\omega \ll 1$ 幂次的双重展开，$G_N E\omega \ll 1$ 控制物体源弯曲时空背景中引力子传播带来的效应，而 $\omega b \ll 1$ 是多极展开参数。原则上，有效场论也可以计算引力子圈带来的量子修正，这类修正被 $\hbar/Eb \ll 1$ 的幂次压低，在天体物理应用中完全可以忽略。由于 $G_N E\omega \ll 1$ ，费曼规则对应平直空间引力耦合世界线上局域化的源。通过幂次计数可以发现，经典极限 $\hbar/Eb \ll 1$ 下，仅存至多包含一个外引力子的图，形式如图 3 所示。得到的费曼积分可以用成熟技术处理 [117]，至少在 $G_N E/b \ll 1$ 阶足够低时，对于任意含时源矩 $I_{a_1\cdots a_\ell}, J_{a_1\cdots a_\ell}$ 都可以解析计算。

It is convenient to compute radiative corrections to binary dynamics, e.g., effects from radiation graviton exchange, directly in the radiation EFT of Eq. (57) rather than in NRGR. The advantage of doing so is that the factorization of contributions from the UV (the multipole moments), which depend on the specific source and IR (radiation), which are universal, is manifest. For astrophysical applications, the relevant quantities are the zero point function, which encodes the radiative corrections to the equations of motion (radiation reaction forces), and the one-point function, which determines the waveform measured at the detector as a function of the time-dependent moments evaluated on the solutions to the PN equations of motion for the orbits.

直接在式 (57) 的辐射有效场论而非 NRGR 中计算双星动力学的辐射修正 (例如辐射引力子交换产生的效应) 更为方便。这样做的优势是，依赖于具体源的紫外贡献 (多极矩) 和具有普适性的红外贡献 (辐射) 的分解是显式的。对于天体物理应用，相关物理量是零点函数和单点函数: 零点函数编码了运动方程的辐射修正 (辐射反作用力)，单点函数则根据在 PN 轨道运动方程解上计算得到的含时多极矩，确定探测器测量到的波形。

In such calculations, one encounters both UV and IR logarithmically divergent Feynman diagrams (The relevant Feynman integrals are isomorphic to those of a fictitious 3D Euclidean field theory of particles with propagators $1/\left(\ell^2-\omega^2\right)$ and (complex valued) "masses" related to the frequencies ω of the external radiation gravitons. Restricting the EFT to the sector with at most one external radiation graviton implies that at most one external momentum can show up in the propagators.), even at the classical level. In order to preserve manifest diff invariance, these are defined via dimensional regularization, where the log divergences correspond to poles in $\varepsilon = 4-d$. We describe the resolution of such IR and UV singularities in the next two sections.

在此类计算中，即便在经典层面也会遇到紫外和红外对数发散的费曼图 (相关费曼积分与一类特殊的三维欧几里得粒子场论的积分同构，该理论具有传播子 $1/\left(\ell^2-\omega^2\right)$，且复“质量”与外辐射引力子的频率 ω 相关。将有效场论限制在最多存在一个外辐射引力子的区域，意味着传播子中最多出现一个外动量。) 为了保持显式微分同胚不变性，这些发散通过维数正规化定义，其中对数发散对应 $\varepsilon = 4-d$ 中的极点。我们将在接下来两节中介绍这类红外和紫外奇点的解决方法。

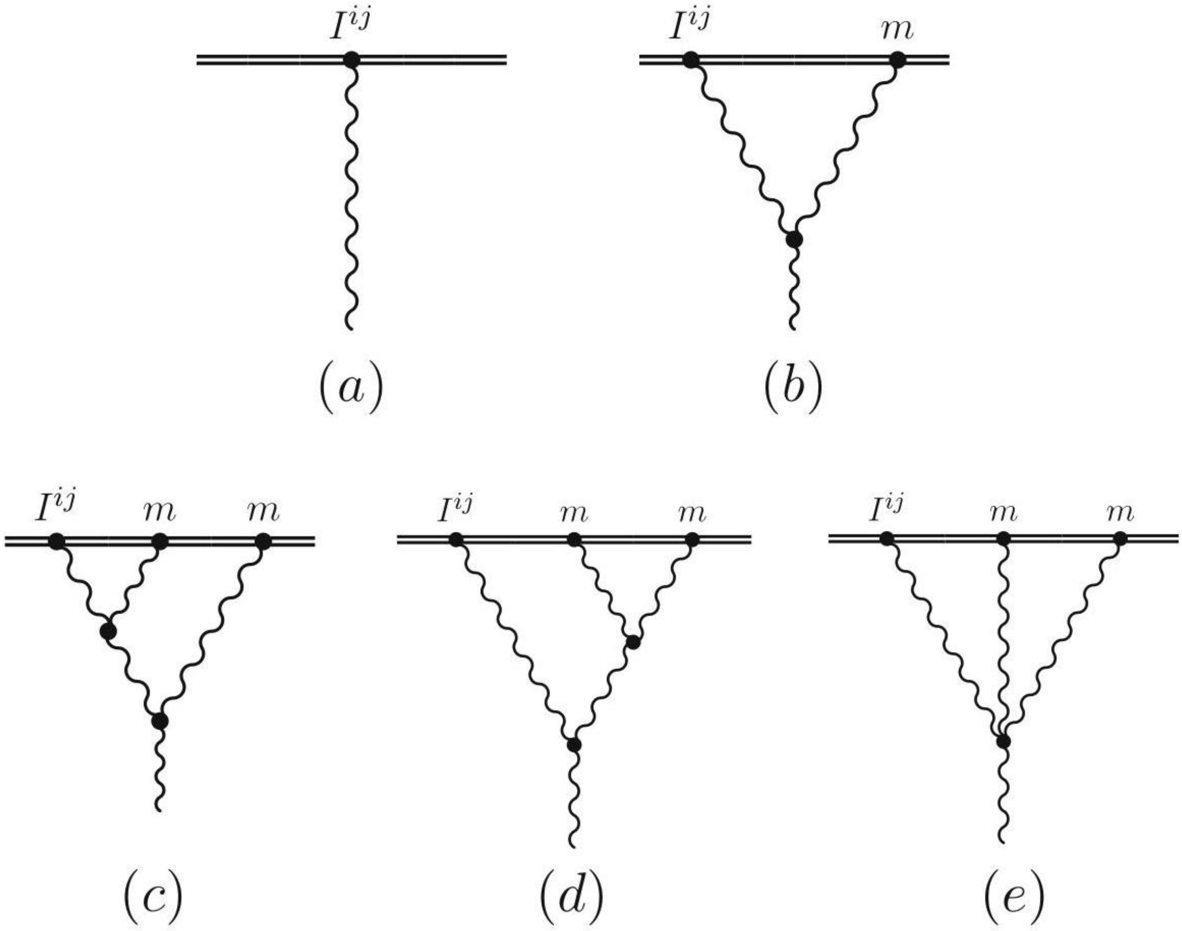


Fig. 3 Leading order quadrupole emission (a) in TSE IR and perturbative ”tail” corrections at $\mathcal{O}(G_N E\omega)$ (b) and $\mathcal{O}(G_N E\omega)^2$ (c)-(e). Diagram (b) has a $1/\varepsilon_{IR}$ singularity in dimensional regularization, while (c)-(e) contain both IR and UV poles in $d = 4-\varepsilon$ spacetime dimensions

图 3 TSE 红外区域中的领头阶四极辐射 (a)，以及 $\mathcal{O}(G_N E\omega)$ 阶 (b) 和 $\mathcal{O}(G_N E\omega)^2$ 阶 (c)-(e) 的微扰“尾巴”修正。图 (b) 在维数正规化中存在 $1/\varepsilon_{IR}$ 奇点，而图 (c)-(e) 在 $d = 4-\varepsilon$ 维时空中同时包含红外极点和紫外极点

Infrared Divergences

红外发散

The IR divergences arise from so-called gravitational wave tails [12], which refer to the distortion of the outgoing graviton wavefunctions by the $1/r$ gravitational potential sourced by the mass monopole, as depicted in Fig. 3b-e. They are analogous to the IR divergences found in non-relativistic Coulomb scattering and in the gravitational context appear first at order $G_N M\omega \ll 1$ beyond leading order radiation emission.

红外发散来源于所谓的引力波尾巴 [12]，指的是出射引力子波函数被质量单极子产生的 $1/r$ 引力势扭曲，如图 3b-e 所示。它们与非相对论库仑散射中发现的红外发散类似，在引力语境下，这类发散首次出现在超越领头阶辐射发射的 $G_N M\omega \ll 1$ 阶。

As an example, consider Fig. 3b, which contains (after tensor reduction into scalar master integrals) a Feynman integral of the form

举个例子，考虑图 3b，该图在张量化约为标量主积分后，包含如下形式的费曼积分

$$\mathbf{q}\overrightarrow{}0\mathcal{A}_{LO}^{\ell=2}(k^0) \times \int_{\mathbf{q}} \frac{1}{\mathbf{q}^2}\frac{1}{2\mathbf{q}\cdot\mathbf{k}} = -\frac{iG_N m\,|k^0|}{\varepsilon_{IR}} \times \mathcal{A}_{LO} + \cdots,$$

(62)where the LO quadrupole emission amplitude Fig. 3 is given by

其中图 3 的领头阶四极辐射发射振幅由下式给出

$$= i\mathcal{A}_{LO}^{\ell=2}(\omega) = \frac{i\omega^2}{4m_{Pl}}\int_{-\infty}^{\infty} dt e^{i\omega t}\varepsilon_{ab}^{*}(k) I^{ab}(t) \tag{63}$$

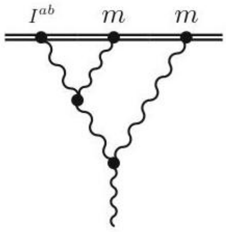
in the CM frame of the composite object, $P^\mu = (m,0,0,0)$ and $x_{\mathrm{CM}}^\mu = 0$.

在复合体系的质心系中，$P^\mu = (m,0,0,0)$ 和 $x_{\mathrm{CM}}^\mu = 0$ 。

Equation (62) has a logarithmic $1/\varepsilon_{IR}$ IR pole when the external graviton goes on-shell, $k_0^2 \to \mathbf{k}^2$, from the region of soft loop momentum $\mathbf{q} \to 0$. Similarly, going next-to-leading order, there is a $1/\varepsilon_{IR}^2$ IR divergence in the diagram Fig. 3c

当外引力子在壳时，方程 (62) 存在一个对数 $1/\varepsilon_{IR}$ 红外极点 $k_0^2 \to \mathbf{k}^2$，源于软圈动量 $\mathbf{q} \to 0$ 区域。类似地，到次领头阶时，图 3c 的费曼图中存在一个 $1/\varepsilon_{IR}^2$ 红外发散

(64)



$$\sim \frac{1}{2!}\left[-\frac{iG_N m|k^0|}{\varepsilon_{IR}}\right]^2 \times \mathscr{A}_{LO}^{\ell=2},$$

when the external graviton goes on-shell, from the region where all the internal momenta go to zero. More generally, at order $(G_N M\omega)^n$, there is $1/\varepsilon_{IR}^n$ pole from the "ladder" diagram containing n internal gravitons, each sourced by a mass monopole, connecting with the radiation graviton emitted by the quadrupole source. In addition, graviton emission in the higher ℓ-th multipole channels receives corrections from n-the order ladder graphs analogous to those shown in Fig. 3.

当外引力子在壳时，发散来源于所有内动量都趋于零的区域。更一般地说，在 $(G_N M\omega)^n$ 阶，包含 n 个由质量单极子产生的内部引力子的“阶梯”图会产生一个 $1/\varepsilon_{IR}^n$ 极点，这些内部引力子连接四极源发射的辐射引力子。此外，更高 ℓ 阶多极通道中的引力子发射会收到来自 n 阶阶梯图的修正，这些阶梯图与图 3 所示的类似。

The resolution [55, 97] of these IR divergences is similar to what happens in QED [114]: in frequency space, the series of $G_N M\omega/\varepsilon_{IR}$ poles exponentiates into an overall phase factor in the graviton emission amplitude [55]. This phase then cancels in the gravitational energy flux (emitted power)

这些红外发散的解决方法 [55, 97] 与量子电动力学中的情况类似 [114]: 在频域中, 一系列 $G_N M\omega/\varepsilon_{IR}$ 极点指数化为引力子发射振幅中的整体相位因子 [55]。该相位随后在引力能流 (辐射功率) 中抵消

$$\frac{d^3 P^\mu}{d\Omega d\omega} = \frac{\omega^2}{8\pi^2} k^\mu |\mathcal{A}(\omega)|^2, \tag{65}$$

where $k^\mu = (\omega, \omega\mathbf{n})$ points from the source in the direction of the gravitational wave detector at $\mathcal{J}^+$. Similarly the flux of angular momentum to infinity, which depends only on $|\mathcal{A}|^2$, is free of IR divergences.

其中 $k^\mu = (\omega, \omega\mathbf{n})$ 从源指向位于 $\mathcal{J}^+$ 处的引力波探测器方向。类似地，仅依赖 $|\mathcal{A}|^2$ 的流向无穷远的角动量流也不存在红外发散。

One can also show that the IR divergences do not affect the waveform seen at infinity: upon transforming the amplitude to the time domain, the IR divergent phase has the effect of shifting the argument of the gravitational wave signal $h(t)$, Eq. (37), recorded at the detector. This shift is arbitrary and is absorbed into the definition of the (experimentally determined) "initial time" when the signal first enters the detector's frequency band [97].

我们还可以证明，红外发散不会影响无穷远处观测到的波形: 将振幅变换到时域后，红外发散相位的作用是平移探测器记录的引力波信号 $h(t)$ 即式 (37) 的自变量。这个平移是任意的，可以被吸收到 (实验确定的)“初始时间”定义中，初始时间即信号刚进入探测器频带的时刻 [97]。

Note that despite the disappearance of the IR regulator $1/\varepsilon_{IR}$ from infrared safe physical observables, the gravitational wave tails leave a measurable imprint on the waveform at $\mathcal{I}^+$. For example, Refs. [7,74] have shown that the entire series of powers of $4\pi G_N M\omega$ in the graviton emission amplitude squared is given by the Sommerfeld factor [106]

需要注意，尽管红外调节器 $1/\varepsilon_{IR}$ 从红外安全的物理可观测量中消失了，但引力波尾巴仍会在 $\mathcal{I}^+$ 处的波形上留下可测量的印记。例如，文献 [7,74] 已经证明，引力子发射振幅模平方中所有 $4\pi G_N M\omega$ 幂次的级数由索末菲因子给出 [106]

$$S(\omega) = 4\pi G_N M\omega / \left(1 - e^{-4\pi G_N M\omega}\right), \tag{66}$$

familiar from Coulomb scattering in non-relativistic quantum mechanics.

这在非相对论量子力学的库仑散射中是我们熟悉的结果。

UV Divergences and Renormalization Group Evolution

紫外发散与重整化群演化

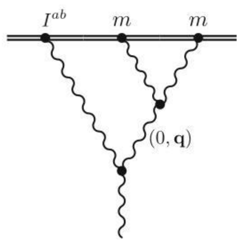
In addition to the usual graviton loop UV divergences of effective quantum gravity [109], suppressed by powers of $\hbar/L \ll 1$, the theory is also afflicted by short-distance singularities which persist even in the classical limit. These classical UV divergences are generic in field theories that are coupled to defects, i.e., Dirac delta function sources of non-zero codimension. Such divergences are resolved by the finite transverse size of the defect in the full theory, but, in the EFT, can be absorbed into local counterterms on the defect worldvolume, generating in some cases nontrivial renormalization group (RG) flows for the Wilson coefficients, even at the classical level [50, 55] .

除了有效量子引力中被 $\hbar/L \ll 1$ 幂次压低的常规引力子圈紫外发散 [109]，该理论还存在即使在经典极限下也依然存在的短距离奇点。这些经典紫外发散在耦合了缺陷 (即余维数非零的狄拉克 δ 函数源) 的场论中普遍存在。这类发散可以通过完整理论中缺陷的有限横向尺寸解决，但在有效场论中，它们可以被吸收进缺陷世界体积的局部抵消项中，在某些情况下甚至会在经典层面也为威尔逊系数产生非平庸的重整化群 (RG) 流 [50, 55] 。

Power UV divergences are generated already at leading in perturbation theory (for instance, from the energy stored in the composite objects $1/r$ Newtonian gravitational field), but dimensional regularization simply defines these to be zero. More interesting logarithmic divergences arise in ZSOR IR starting at order $(G_N M\omega)^2$ in the expansion. They appear, for instance, in the order $(G_N M\omega)^2$ corrections to graviton emission in the mass quadrupole channel, as in Fig. 3c-e. For example,

幂次紫外发散已经在微扰论领头阶产生 (例如来自复合对象 $1/r$ 牛顿引力场储存的能量)，但维数正规化直接将这类发散定义为零。更有趣的对数发散在 ZSOR 红外区域出现，从展开的 $(G_N M\omega)^2$ 阶开始。例如它们出现在质量四极矩通道引力子辐射的 $(G_N M\omega)^2$ 阶修正中，如图 3c-e 所示。举例来说，

$$
\begin{aligned}
&\sim \mathcal{A}_{LO}^{\ell=2}\left(k^0\right)\times\int_{\mathbf{q}}\frac{1}{|\mathbf{q}|}\frac{1}{k_0^2-(\mathbf{k}+\mathbf{q})^2}\mathbf{q}\to\infty\mathcal{A}_{LO}^{\ell=2}\left(k^0\right)\times\int_{\mathbf{q}}\frac{1}{|\mathbf{q}|^3}\\
&\sim\frac{\left(G_N m k^0\right)^2}{\varepsilon_{UV}}\times\mathcal{A}_{LO}^{\ell=2}\left(k^0\right),
\end{aligned}
\tag{67}
$$



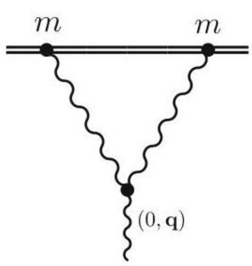
has a $1/\varepsilon_{UV}$ pole from the $|\mathbf{q}|\gg k^0$ region of loop momentum.

在圈动量的 $|\mathbf{q}|\gg k^0$ 区域产生一个 $1/\varepsilon_{UV}$ 极点。

Physically, the log divergence in Eq. (67) is generated by the propagation of the emitted radiation mode in the short-distance part of the source's static gravitational potential, namely, the order $(G_N M/r)^2$ relativistic correction to the metric sourced by the mass monopole in Eq. (57). This singular $\sim 1/r^2$ potential is an artifact of the EFT. It begins to dominate at distance scales where the multipole expansion is no longer a good description of the internal structure of the binary in the full theory. For example, in the diagram in Fig. 3d, the Fourier transform of the $1/r^2$ potential is embedded in the subdiagram

物理上，式 (67) 中的对数发散是由源静态引力势的短距离部分中辐射模式的传播产生的，也就是式 (57) 中质量单源诱导度规的 $(G_N M/r)^2$ 阶相对论修正。这个奇异的 $\sim 1/r^2$ 势是有效场论的人为产物。它在距离尺度足够小的时候开始占据主导，此时多极展开已经无法很好描述完整理论中双星的内部结构。例如，在图 3d 的图中，$1/r^2$ 势的傅里叶变换嵌入在子图中

(68)



$$\sim\frac{1}{|\mathbf{q}|},$$

which accounts for the factor $1/|\mathbf{q}|$ in the Feynman integral over the momentum flowing out of the quadrupole vertex in Fig. 3d.

这就解释了图 3d 中流出四极顶点动量的费曼积分里的 $1/|\mathbf{q}|$ 因子。

The sum of the diagrams in Figs. 3c-e contains a UV divergent (The sum of Figs. 3c-e also contains $1/\varepsilon_{IR}$ and $1/\varepsilon_{IR}^2$ singularities, as discussed in the previous section. It is possible [55] to use the method of regions [9] to disentangle the UV and IR contributions to the coefficient of the $1/\varepsilon$ pole.) term

图 3c-e 的图求和后包含一个紫外发散项 (正如上一节讨论的，图 3c-e 的求和还包含 $1/\varepsilon_{IR}$ 和 $1/\varepsilon_{IR}^2$ 奇点。可以 [55] 利用区域法 [9] 分离出 $1/\varepsilon$ 极点系数中的紫外和红外贡献。)

$$\frac{1046}{315}\frac{(G_N M\omega)^2}{\varepsilon_{UV}} \times \mathcal{A}_{LO}^{\ell=2}(\omega) \tag{69}$$

which is analytic in the frequency of the emitted graviton and can be absorbed into a local counterterm on the zoomed out object's worldline. Specifically, the UV pole renormalizes the electric quadrupole Wilson coefficient $I_{ab}(\tau)$ in ZSO IR.

该项在发射引力子的频率处是解析的，可以被吸收进放大后物体世界线的局部抵消项中。具体来说，紫外极点对 ZSO 红外中电四极威尔逊系数 $I_{ab}(\tau)$ 做重整化。

The graviton emission matrix element is finite when expressed in terms of the renormalized quadrupole, at the expense of introducing an arbitrary renormalization scale μ . Explicit logarithmic dependence on μ arising from the evaluation of the dimensionally regularized Feynman integrals cancels against the subtraction scale dependence of the renormalized moment $I_{ab}(\mu, \tau)$, ensuring that the emission amplitude (a physical observable) is insensitive to the choice of μ . This requires that, in frequency space, the electric quadrupole satisfies the RG equation (RGE) [55]:

用重整化后的四极矩表示时，引力子辐射矩阵元是有限的，代价是引入了任意的重整化标度 μ 。维数正规化费曼积分计算产生的对 μ 的显式对数依赖，会抵消重整化矩 $I_{ab}(\mu, \tau)$ 的减除标度依赖，保证辐射振幅 (一个物理可观测量) 不依赖 μ 的选择。这要求在频率空间中，电四极矩满足重整化群方程 (RGE) [55]:

$$\mu \frac{d}{d\mu} I_{ab}(\omega, \mu) = -\frac{214}{105}(G_N M\omega)^2 I_{ab}(\omega, \mu) . \tag{70}$$

The RGE is universal, so it can be used to predict the pattern of logarithms of the frequency ω in the matrix element for the emission of soft graviton radiation from any localized source of finite size. By running Eq. (70) from $\mu_{UV} \sim 1/b$ to $\mu_{IR} \sim \omega$, it is possible to "resume" the series of powers $(G_N m\omega)^{2n} \ln^n(\omega b)$ in quadrupole emission. Similar RG flows to Eq. (70) occur for multipole moments beyond $\ell = 2$, [4, 47, 56] , so all terms of the form $(G_N m\omega)^n \ln^m \omega b$ induced by soft graviton radiation are in principle known.

RGE 具有普适性，因此它可用于预测任意有限尺寸局域源发出软引力子辐射的矩阵元中频率 ω 对数的分布形式。通过将式 (70) 从 $\mu_{UV} \sim 1/b$ 跑动到 $\mu_{IR} \sim \omega$ ，可以对四极辐射中的幂级数 $(G_N m\omega)^{2n} \ln^n(\omega b)$ 进行“重求和”。超出 $\ell = 2$, [4, 47, 56] 的多极矩也存在类似式 (70) 的 RG 流，因此软引力子辐射诱导的所有 $(G_N m\omega)^n \ln^m \omega b$ 形式项原则上都是已知的。

As an application of Eq. (70), we can use it to predict the pattern of PN logarithms in the $\ell = 2$ electric radiated power from a non-relativistic binary, by inserting the renormalized quadrupole into Eq. (63) and running the RG from the UV at a scale $\mu_{UV} \sim 1/r$ where the binary matches onto ZSe IR down to the IR scale $\mu_{IR} \sim \omega$. For example, the entire series of integer powers $\left(v^6 \ln v\right)^n$ in the energy flux of $\ell = 2$ radiation from a binary in a circular orbit with velocity $v \ll 1$, normalized to the leading order quadrupole radiation formula,

作为式 (70) 的一个应用，我们可以用它预测非相对论双星 $\ell = 2$ 电辐射功率中 PN 对数的分布形式：将重整化后的四极矩代入式 (63)，令 RG 从紫外标度 $\mu_{UV} \sim 1/r$ (此处双星匹配到 ZSe IR) 跑动到红外标度 $\mu_{IR} \sim \omega$ 即可。例如，对于速度为 $v \ll 1$ 的圆轨道双星，其 $\ell = 2$ 辐射能流中按领头阶四极辐射公式归一化的整个整数幂级数 $\left(v^6 \ln v\right)^n$ ，

$$\frac{\dot{E}^{\ell=2E}_{\log}}{\dot{E}^{\ell=2E}_{\mathrm{LO}}} = \left[\frac{\mu}{\mu_0}\right]^{-\frac{428}{105}(G_N M\omega)^2} = 1 - \frac{428}{105} v^6 \ln v + \frac{91592}{11025} v^{12} \ln^2 v - \frac{39201376}{347287} v^{18} \ln^3 v + \cdots, \tag{71}$$

is fully determined by RG evolution.

完全由 RG 演化确定。

EFT reasoning predicts that the same pattern of logarithms should also appear in other kinematic regimes. For example, consider binary black holes in the ”EMRI” limit of hierarchical masses, so that the smaller constituent can be treated as a perturbation of the Kerr geometry sourced by the heavier one. A semi-analytic treatment [45] of the energy flux from non-circular $v \ll 1$ orbits around a black hole up to 14PN (!) order found in logarithmic terms that match those predicted by Eq. (71). The fact that the non-analytic low-frequency behavior of waves propagating in curved spacetime can be obtained from $1/\epsilon_{UV}$ poles of Feynman diagrams in flat spacetime provides a sharp example of the universality of the worldline EFT predictions.

EFT 推导预测，相同的对数分布也应当出现在其他运动学区域中。例如，考虑质量分层的“EMRI”极限下的双黑洞系统，此时较小的天体可被视为大质量天体产生的克尔几何的微扰。对黑洞周围非圆 $v \ll 1$ 轨道的能流进行到 14PN(!) 阶的半解析分析 [45]，发现其中的对数项与式 (71) 的预测一致。弯曲时空中传播的波的非解析低频行为可以从平直时空费曼图的 $1/\epsilon_{UV}$ 极点得到，这一事实给出了世界线 EFT 预测普适性的鲜明例证。

While the RG can efficiently generate the coefficients of logarithms, by itself, it cannot fix the precise UV scale $\mu_{UV} \sim 1/r$ where we define the Wilson coefficients. That must be determined by performing a matching calculation to the more UV complete theory that resolves the internal structure of the radiating object. For nonrelativistic applications, with $G_N M\omega \sim v^3$, a 3PN matching calculation to NRGR is needed to fix the relation between the renormalized moments at $\mu_{UV} \sim 1/r$ and the microscopic (orbital) degrees of freedom of the binary system. The basic procedure for matching TSE IR to NRGR is the subject of the next section.

虽然 RG 可以高效得到对数项的系数，但它本身无法确定我们定义威尔逊系数时所用的精确紫外标度 $\mu_{UV} \sim 1/r$ 。该标度必须通过对能解析辐射天体内部结构的更紫外完备的理论做匹配计算才能得到。对于非相对论应用，当 $G_N M\omega \sim v^3$ 时，需要通过与 NRGR 做 3 阶 PN 匹配计算，才能确定 $\mu_{UV} \sim 1/r$ 处重整化矩与双星系统的微观 (轨道) 自由度之间的关系。将 TSE IR 匹配到 NRGR 的基本流程是下一节的主题。

Matching NRGR to 750 IR

将 NRGR 匹配到 750 IR

Focusing now on non-relativistic binaries, we assign power counting $I_{a_1\cdots a_\ell} \sim Mr^\ell$, $J_{a_1\cdots a_\ell} \sim Mvr^\ell$, in which case the two expansion parameters of TSE IR control effects which are down by different powers in v : multipole corrections in powers of $\omega r \sim v$ and gravitational wave tails such as those in Fig. 3 in powers of $G_N M\omega \sim v^3$. In the PN regime, it is possible to fix the Wilson coefficients in Eq. (57) by matching to NRGR at distance scales $\gtrsim r$ where the two theories are both valid.

现在我们聚焦非相对论双星，给出幂次计数规则 $I_{a_1\cdots a_\ell} \sim Mr^\ell$ 、$J_{a_1\cdots a_\ell} \sim Mvr^\ell$ ，在此情况下 TSE IR 的两个展开参数控制了 v 中不同阶的压低效应: $\omega r \sim v$ 幂次的多极修正，以及 $G_N M\omega \sim v^3$ 幂次的引力波拖尾 (如图 3 所示)。在后牛顿区域，我们可以通过在两套理论都成立的距离尺度 $\gtrsim r$ 上与 NRGR 匹配，来固定式 (57) 中的威尔逊系数。

Since the potential modes in NRGR do not go on-shell, the effective Lagrangian for the radiation field coupled to the worldline degrees of freedom $\mathbf{x}_{1,2}$ (and the spins $\mathbf{S}_{1,2}$ which we have been largely ignoring) is local at length scales larger than the orbital scale r . Formally, we can obtain this effective Lagrangian by integrating out $H_{\mathbf{p};\mu\nu}$

由于 NRGR 中的势模不会在壳，耦合到世界线自由度 $\mathbf{x}_{1,2}$ (以及我们基本忽略的自旋 $\mathbf{S}_{1,2}$) 的辐射场有效拉格朗日量在大于轨道尺度 r 的长度尺度上是定域的。形式上我们可以通过积去 $H_{\mathbf{p};\mu\nu}$ 得到该有效拉格朗日量

$$e^{iS_{\sum I_R}[\mathbf{x}_{1,2},\bar{g}]} = \int \mathcal{D}H_{\mathbf{p};\mu\nu} e^{iS_{NRGR}}, \tag{72}$$

without the need to impose in-in boundary conditions. The long-distance theory can be organized as an expansion in powers of the radiation field

无需引入入入边界条件。长程理论可以按辐射场的幂次展开组织

$$S_{Z_{ER}} = S_{EH}[\bar{g}] + \Gamma^{(0)}[\mathbf{x}_{1,2}] + \Gamma^{(1)}[\mathbf{x}_{1,2}, \bar{h}] + \cdots, \tag{73}$$

where $\Gamma^{(0)}[\mathbf{x}_{12}]$ is independent of $\bar{h}_{\mu v}$, while $\Gamma^{(1)}$ depends linearly

其中 $\Gamma^{(0)}[\mathbf{x}_{12}]$ 与 $\bar{h}_{\mu v}$ 无关，而 $\Gamma^{(1)}$ 呈线性依赖

$$\Gamma^{(1)}\left[\mathbf{x}_{1,2}, \bar{h}\right] = -\frac{1}{2m_{\text{Pl}}} \int d^4 x \overline{\tau}^{\mu\upsilon}(x)\, \bar{h}_{\mu\upsilon}. \tag{74}$$

for some function $\overline{\tau}^{\mu\upsilon}(x)$ of the orbital degrees of freedom.

它是轨道自由度的某个函数 $\overline{\tau}^{\mu\upsilon}(x)$ 。

In perturbation theory, $\overline{\tau}^{\mu\upsilon}(x)$ can be identified with the sum over Feynman diagrams that only carry internal potential graviton lines and a single external radiation graviton. We perform the path integral over $H_{\mathbf{p};\mu\nu}$ in background field gauge [3, 31] , in which the gauge fixing term is invariant under diffeomorphisms $\delta\bar{h}_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$ acting on the background filed. This implies in particular the conservation law $\partial_\nu \overline{\tau}^{\mu\nu} = 0$. For applications to classical binary inspirals, it is only necessary to match to NRGR in the sectors with zero or one external radiation gravitons, as subsequent emissions bring down powers of $\hbar/L \to 0$. By diffeomorphism invariance, knowing the vacuum and single-graviton matrix elements in Eq. (73) is sufficient to determine the relevant nonlinear couplings of the radiation mode as well.

微扰论中，$\overline{\tau}^{\mu\upsilon}(x)$ 可以对应所有仅含内部势引力子线和一条外部辐射引力子线的费曼图之和。我们在背景场规范 [3, 31] 下对 $H_{\mathbf{p};\mu\nu}$ 做路径积分，该规范下规范固定项在作用于背景场的微分同胚 $\delta\bar{h}_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$ 下不变，这尤其给出守恒律 $\partial_\nu \overline{\tau}^{\mu\nu} = 0$ 。对于经典双星并合的应用，仅需要匹配 NRGR 中零个或一个外部辐射引力子的 sector，因为后续辐射会带来 $\hbar/L \to 0$ 阶的压低。由微分同胚不变性，只要知道式 (73) 中的真空和单引力子矩阵元，就足以确定辐射模所有相关的非线性耦合。

Two-Body Potentials

两体势

In NRGR, the term $\Gamma^{(0)}[\mathbf{x}_{12}]$ corresponds to Feynman diagrams involving arbitrary insertions of potential and worldline vertices, but no internal or external radiation lines. As such, it is a functional only of the worldline variables and therefore defines the conservative dynamics of the two-body system, in the sense that the Euler-Lagrange equations for $\Gamma^{(0)}[\mathbf{x}_{12}] = \int dt L\left[\mathbf{x}_{1,2}\right]$, with

在 NRGR 中，项 $\Gamma^{(0)}[\mathbf{x}_{12}]$ 对应费曼图，其中包含任意数量的势和世界线顶点插入，但没有内部或外部辐射线。因此，它仅为世界线变量的泛函，由此定义了两体系统的保守动力学，即 $\Gamma^{(0)}[\mathbf{x}_{12}] = \int dt L\left[\mathbf{x}_{1,2}\right]$ 的欧拉-拉格朗日方程满足

$$L\left[\mathbf{x}_{1,2}\right] = -\sum_A m_A \sqrt{1 - \mathbf{v}_A^2} + \Delta L\left[\mathbf{x}_{1,2}\right], \tag{75}$$

define the equations of motion for the trajectories $\mathbf{x}_{1,2}$ including all possible short-distance gravitational interactions, but neglecting radiation (radiation reaction effects will be discussed below in section ”Radiation Reaction in Tab IR”). For example, using the integral $\int_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}}/\mathbf{p}^2 = (4\pi\,|\mathbf{x}|)^{-1}$ as well as the definition $m_{\text{Pl}}^{-2} = 32\pi G_N$, the leading order term,

可以得到轨迹 $\mathbf{x}_{1,2}$ 的运动方程，该方程包含所有可能的短程引力相互作用，但忽略了辐射 (辐射反作用效应将在下文“切向红外区的辐射反作用”一节讨论)。例如，利用积分 $\int_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}}/\mathbf{p}^2 = (4\pi\,|\mathbf{x}|)^{-1}$ 以及定义 $m_{\mathrm{Pl}}^{-2} = 32\pi G_N$，领头阶项

$$
\begin{array}{c} 1 \\ 1 \end{array}
$$

$$
\xrightarrow[\substack{\int_1^1 \rho \\ t^2}]{} \xrightarrow{\int_1^1 \rho} = \int dt_1 dt_2 \int_{\mathbf{p},\mathbf{q}} \left(-\frac{im_1}{2m_{\mathrm{Pl}}} e^{i\mathbf{p}\cdot\mathbf{x}_1(t_1)}\right)\left(-\frac{im_2}{2m_{\mathrm{Pl}}} e^{i\mathbf{q}\cdot\mathbf{x}_2(t_2)}\right)
$$

$$
\times \langle H_{\mathbf{p};00}(t_1) H_{\mathbf{q};00}(t_2)\rangle = \frac{im_1 m_2}{4m_{\mathrm{Pl}}^2} P_{00,00} \int dt \int_{\mathbf{p}} \frac{e^{i\mathbf{p}\cdot\mathbf{x}_{12}}}{\mathbf{p}^2}
$$

$$
= i \int dt \frac{G_N m_1 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|} = i\Delta L_{0PN},
$$

reproduces the Newtonian gravitational interaction, as one would hope.

恰好可以重现牛顿引力相互作用，符合预期。

Similarly, the sum of the diagrams Fig. 4 yields the so-called Einstein-Infeld-Hoffmann (EIH) correction to non-relativistic gravitational motion

类似地，对图 4 中的所有费曼图求和，即可得到非相对论引力运动的所谓爱因斯坦-因费尔德-霍夫曼 (EIH) 修正

$$
\begin{aligned}
\Delta L_{1PN} = L_{EIH} = \frac{1}{8}\sum_A m_A \mathbf{v}_A^4 \\
+\frac{G_N m_1 m_2}{2\,|\mathbf{x}_1 - \mathbf{x}_2|}\left[3\left(\mathbf{v}_1^2 + \mathbf{v}_2^2\right) - 7\mathbf{v}_1\cdot\mathbf{v}_2 - \frac{(\mathbf{v}_1\cdot\mathbf{x}_{12})(\mathbf{v}_2\cdot\mathbf{x}_{12})}{\mathbf{x}_{12}^2}\right] \\
-\frac{G_N^2 m_1 m_2 (m_1 + m_2)}{2|\mathbf{x}_1 - \mathbf{x}_2|^2}
\end{aligned}
\tag{76}
$$

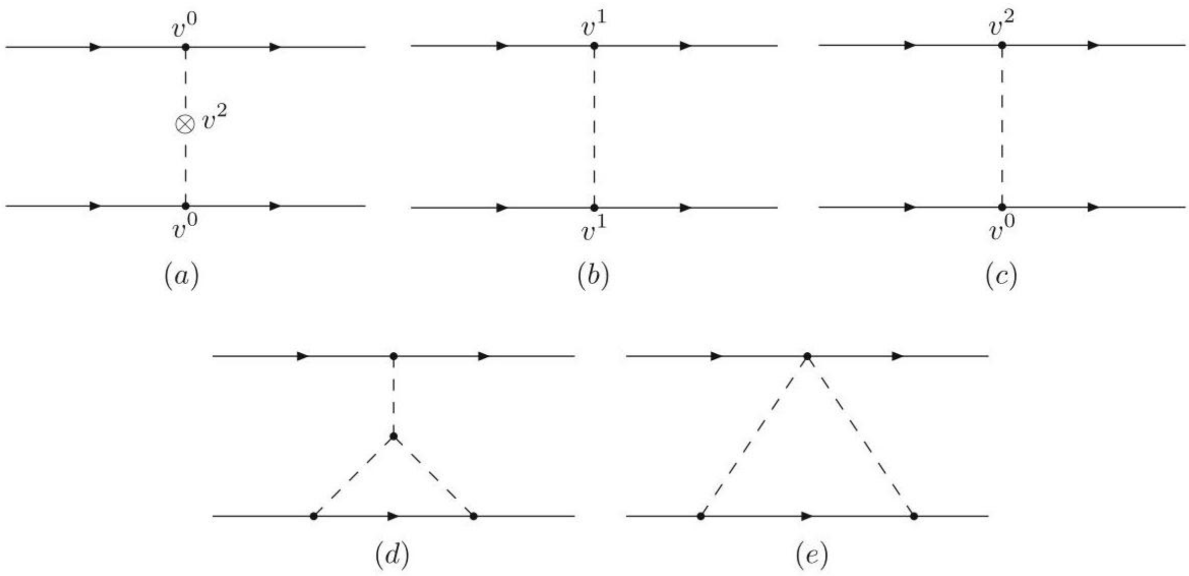


Fig. 4 1PN corrections to conservative two-body dynamics. In addition to the diagrams shown, there are also mirror ($1 \leftrightarrow 2$) diagrams to (**c**) , (**d**) ,(e) which also contribute at $\mathcal{O}\left(v^2\right)$

图 4 保守两体动力学的 1PN 修正。除图中所示图外，(**c**) , (**d**) 、(e) 还存在镜像 ($1 \leftrightarrow 2$) 图，它们同样对 $\mathcal{O}\left(v^2\right)$ 阶有贡献

where we have also included the first relativistic correction to the kinetic energy of the particles, from expanding the first term in Eq. (75). The EIH Lagrangian has implications for planetary motion in the solar system, so that, e.g., the perihelion advance of Mercury's orbit can be regarded as a probe of the cubic self-interaction vertex of the graviton; see Fig. 4d.

其中我们还纳入了对粒子动能的一阶相对论修正，该修正来自对式 (75) 第一项的展开。EIH 拉格朗日量对太阳系的行星运动有影响，因此水星轨道的近日点进动可作为引力子三次自相互作用顶点的探针；参见图 4d。

For two-body systems, at higher PN orders, one encounters momentum space Feynman integrals that are formally the same that would appear in the radiative corrections to propagators (two-point functions) in a massless Euclidean QFT living in $3-\varepsilon$ spatial dimensions. Such multi-loop integrals are tractable by standard techniques of perturbative QFT (see [105, 117] for comprehensive reviews). In a generic gauge, the potentials at the nPN order require the evaluation of n -loop Feynman integrals, but by exploiting a convenient field redefinition of the graviton that is well suited to the non-relativistic limit, introduced in Refs. [75-77], it is possible to postpone the number of loops by one order in perturbation theory. Within the EFT approach, the non-relativistic spin-independent potentials at 2PN order were first tackled in Ref. [49], which introduced some of the tools necessary to carry out higher-order PN loop diagrams. The systematic study of higher-order spinless PN potentials was initiated in [40] and extended [41,43,44] to 4PN in [44], which is the current state-of the-art in NRGR.

对于两体系统，在更高后牛顿 (PN) 阶会遇到动量空间费曼积分，这些积分在形式上与 $3-\varepsilon$ 维空间中无质量欧几里得量子场论 (QFT) 传播子 (两点函数) 辐射修正中出现的积分完全相同。这类多圈积分可以用微扰量子场论的标准方法处理 (综合综述参见文献 [105, 117])。在一般规范下，nPN 阶的势需要计算 n 圈费曼积分，但利用参考文献 [75-77] 中引入的、适配非相对论极限的引力子场重定义，可以将圈数在微扰论中降低一阶。在有效场论 (EFT) 框架下，2PN 阶非相对论无自旋两体势的研究最早由文献 [49] 完成，该工作引入了处理高阶 PN 圈图所需的部分工具。无自旋高阶 PN 势的系统研究始于文献 [40]，并经文献 [41,43,44] 推广至 4PN 阶，这是目前 NRGR 领域的最新研究进展。

Radiation and Multipole Moments

辐射与多极矩

In light of Eq. (72), the function $\bar{\tau}^{\mu\nu}$ can be calculated by summing all Feynman diagrams in NRGR containing only internal potential lines and a single off-shell external radiation line; see Fig.5. Even though, in background field gauge, $\bar{\tau}^{\mu\nu}$ is conserved, $\partial_{\nu}\bar{\tau}^{\mu\nu}=0$, it should not be confused with the energy-momentum pseudotensor $\tau^{\mu\nu}$ of the entire system, which receives both radiative and potential contributions. Nevertheless, the total four-momentum of the composite system can be calculated directly from $\bar{\tau}^{\mu\nu}$

根据式 (72)，函数 $\bar{\tau}^{\mu\nu}$ 可通过对 NRGR 中所有仅含内线势线和单条离壳外辐射线的费曼图求和得到；参见图 5。尽管背景场规范下 $\bar{\tau}^{\mu\nu}$ 守恒，$\partial_{\nu}\bar{\tau}^{\mu\nu}=0$，但它不应与整个系统的能量动量赝张量 $\tau^{\mu\nu}$ 混淆，后者同时包含辐射和势贡献。不过，复合系统的总四动量可直接由 $\bar{\tau}^{\mu\nu}$ 计算得到

$$P^{\mu}=\int d^{d-1}\mathbf{x}\bar{\tau}^{0\mu}(x)=\int d^{d-1}\mathbf{x}\tau^{0\mu}(x), \tag{77}$$

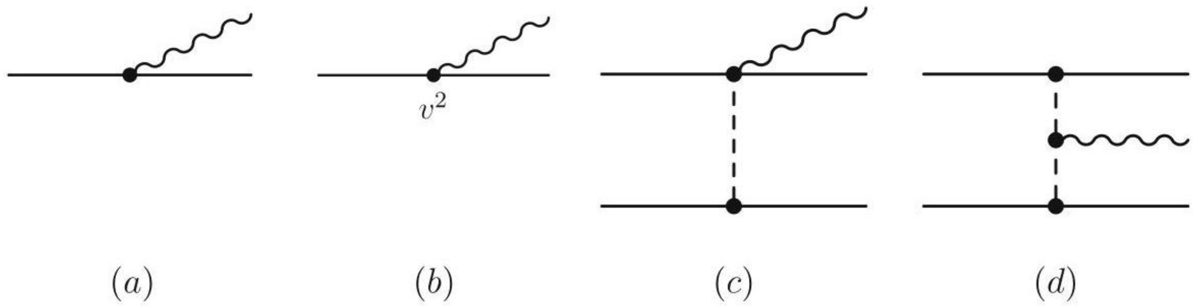


Fig. 5 Diagrammatic expansion of $\bar{\tau}^{\mu\nu}$ in NRGR. (a) is order 0PN (b)-(d) are 1PN

图 5 NRGR 中 $\bar{\tau}^{\mu\nu}$ 的图展开。(a) 为 0PN 阶，(b)-(d) 为 1PN 阶

since radiative corrections in ZSO IR do not renormalize (In dimensional regularization, where scaleless momentum space integrals are defined to be zero) the static part of $\tau^{\mu\nu}(k)=\int d^4xe^{ik\cdot x}\tau^{\mu\nu}(x)$, i.e., the part proportional to a Dirac delta function of the external graviton frequency k^0 . Similarly, the system’s center of mass worldline

这是因为 ZSO 红外区域的辐射修正不会对 $\tau^{\mu\upsilon}(k) = \int d^4xe^{ik\cdot x}\tau^{\mu\upsilon}(x)$ 的静态部分 (即与外部引力子频率 k^0 的狄拉克 δ 函数成正比的部分，维数正则化中标度动量空间积分定义为零) 做重整化。同理，系统的质心世界线

$$\mathbf{X}_{\mathrm{CM}}\left(x^0\right) = \frac{\int d^{d-1}\mathbf{x}\mathbf{x}\tau^{00}\left(x^0, \mathbf{x}\right)}{P^0} \tag{78}$$

does not get renormalized by radiation and may be evaluated by replacing $\tau^{\mu 0} \mapsto \overline{\tau}^{\mu 0}$ inside the integral. Because $\overline{\tau}^{\mu\upsilon}$ is conserved, the CM coordinate drifts with uniform velocity $d\mathbf{X}_{\mathrm{CM}}/dx^0 = \mathbf{P}/P^0$ in any asymptotic Lorentz frame.

不会因辐射发生重整化，可通过替换积分内的 $\tau^{\mu 0} \mapsto \overline{\tau}^{\mu 0}$ 得到。由于 $\overline{\tau}^{\mu\upsilon}$ 守恒，任意渐近洛伦兹系中质心坐标都会以匀速 $d\mathbf{X}_{\mathrm{CM}}/dx^0 = \mathbf{P}/P^0$ 漂移

If $\overline{\tau}^{\mu\nu}$ is given, one can extract the moments in τ_{CIR} by inserting Eq. (54) into Eq. (73) and decomposing the coefficients into representations of the $SO(3)$ that preserves the CM four-momentum of the composite system. It is convenient to do this in the CM frame $u^\mu = P^\mu/P^0 = (1, \mathbf{0})$ and $\mathbf{X}_{\mathrm{CM}} = 0$. In this frame, the leading term in the Taylor expansion

已知 $\overline{\tau}^{\mu\nu}$ 后，我们可以将式 (54) 代入式 (73)，再将系数分解到保持复合系统质心四动量的 $SO(3)$ 表示中，从而提取出 τ_{CIR} 中的矩。在质心系 $u^\mu = P^\mu/P^0 = (1, \mathbf{0})$ 和 $\mathbf{X}_{\mathrm{CM}} = 0$ 中计算尤为方便。在此参考系中，泰勒展开的领头项

$$-\frac{1}{2m_{\mathrm{Pl}}}\int dx^0 \left[\int d^{d-1}\mathbf{x}\overline{\tau}^{\mu\upsilon}\left(x^0, \mathbf{x}\right)\right]\bar{h}_{\mu\upsilon}\left(x^0, 0\right), \tag{79}$$

receives contributions from terms with $\mu\upsilon = 00, ij$ only.

仅由含 $\mu\upsilon = 00, ij$ 的项贡献。

By Eq. (60), the $\mu\upsilon = 00$ part is supposed to match the expansion of the term $\int d\tau L(X, x(\tau), \bar{g}) \subset S_{T\otimes\Pi}$ in the EFT to linear order in $\bar{h}_{\mu\nu}$, i.e.,

根据式 (60)，$\mu\upsilon = 00$ 部分应当匹配有效场论中 $\int d\tau L(X, x(\tau), \bar{g}) \subset S_{T\otimes\Pi}$ 项在 $\bar{h}_{\mu\nu}$ 一阶下的展开，即:

$$H_X = \int d^{d-1}\mathbf{x}\overline{\tau}^{00}. \tag{80}$$

On the other hand, the $\mu\upsilon = ij$ part of Eq. (79) does not contribute at leading order in the multipole expansion, since the conservation law $\partial_\upsilon\overline{\tau}^{\mu\upsilon} = 0$, implies ”moment relations” identical to those obeyed by the full pseudotensor (see any textbook on gravitational radiation, e.g., [85,115]):

另一方面，式 (79) 的 $\mu\upsilon = ij$ 部分在多极展开的领头阶没有贡献，因为守恒律 $\partial_\upsilon\overline{\tau}^{\mu\upsilon} = 0$ 给出了与完整赝张量满足的完全相同的“矩关系”(参见任意引力辐射教材，例如 [85,115]):

$$\int d^{d-1}\mathbf{x}\bar{\tau}^{ij}\left(x^0,\mathbf{x}\right)=\frac{1}{2}\frac{d^2}{dx^{0^2}}\int d^{d-1}\mathbf{x}\bar{\tau}^{00}\left(x^0,\mathbf{x}\right)x^ix^j. \tag{81}$$

Therefore, after integrating by parts, the $\bar{\tau}^{ij}\bar{h}_{ij}$ term in Eq. (79) is proportional on-shell to $\partial_0^2\bar{h}_{ij}=\nabla^2\bar{h}_{ij}$, which is order $\ell=2$ in the multipole expansion.

因此，分部积分后，式 (79) 中的 $\bar{\tau}^{ij}\bar{h}_{ij}$ 项在壳上正比于 $\partial_0^2\bar{h}_{ij}=\nabla^2\bar{h}_{ij}$ ，在多极展开中为 $\ell=2$ 阶。

In a general frame, there are two independent $SO(3)$ moments at order $\ell=1$, both of which are contained in the decomposition of the term

在任意一般参考系中，$\ell=1$ 阶存在两个独立的 $SO(3)$ 矩，二者都包含在该项的分解中

$$-\frac{1}{2m_{\mathrm{Pl}}}\int dx^0\left[\int d^3\mathbf{x}\bar{\tau}^{\mu\upsilon}\left(x^0,\mathbf{x}\right)x^j\right]\partial_j\bar{h}_{\mu\upsilon}\left(x^0,0\right) \tag{82}$$

in Eq. (74). The electric dipole moment is simply the CM coordinate, contained in the $\mu\upsilon=00$ part

见式 (74)。电偶极矩就是质心坐标，它包含在 $\mu\upsilon=00$ 部分中

$$-\frac{1}{2m_{\mathrm{Pl}}}P^0\int dx^0X_{\mathrm{CM}}^i\partial_i\bar{h}_{00}. \tag{83}$$

which we set to zero by our choice of coordinates. To extract the magnetic dipole, project the $\mu\upsilon=0i$ part of Eq. (82) into even or odd parts under permutations $i\leftrightarrow j$. The symmetric part

通过我们的坐标选择将其设为零。为提取磁偶极矩，将式 (82) 的 $\mu\upsilon=0i$ 部分投影到置换 $i\leftrightarrow j$ 下的偶部或奇部。对称部分

$$\int d^3\mathbf{x}x^{(i)\bar{\tau}j|0}=\frac{1}{2}\int d^3\mathbf{x}\partial_k\left(x^ix^j\right)\bar{\tau}^{0k}=\frac{1}{2}\frac{d}{dx^0}\int d^3\mathbf{x}x^ix^j\bar{\tau}^{00}, \tag{84}$$

is, after integrating parts and using the conservation of $\bar{\tau}^{\mu\nu}$, higher order in the multipole expansion. Likewise the various projections of $\int d^3\mathbf{x}\bar{\tau}^{ij}x^k$ onto irreps of the permutation group do not generate moments with $\ell<2$. This leaves behind a coupling of the form

分部积分并利用 $\bar{\tau}^{\mu\nu}$ 守恒后，是多极展开中的更高阶项。同理，将 $\int d^3\mathbf{x}\bar{\tau}^{ij}x^k$ 向置换群不可约表示做各类投影也不会得到含 $\ell<2$ 的矩。这仅留下形式为

$$-\frac{1}{4m_{\mathrm{Pl}}}\int dx^0L^{ij}\left(\partial_i\bar{h}_{0j}-\partial_j\bar{h}_{0i}\right), \tag{85}$$

to the angular momentum of the system, i.e., the antisymmetric moment

与系统角动量的耦合，即反对称矩

$$L^{ij}=\int d^3\mathbf{x}\left(x^i\tau^{0j}-x^j\tau^{0i}\right). \tag{86}$$

(As for P^μ and $\mathbf{X}_{\rm CM}$, we can replace $\overline{\tau}^{0\mu} \mapsto \tau^{0\mu}$ in this expression in dimensional regularization.)

(至于 P^μ 和 $\mathbf{X}_{\rm CM}$ ，我们可以在维数正规化中替换此式中的 $\overline{\tau}^{0\mu} \mapsto \tau^{0\mu}$ 。)

Both $\ell = 1$ moments fit into the same coupling $-\int d\tau S^{ab}\Omega_{ab}$ in the worldline EFT. To match to the worldline, we have chosen the comoving frame to be trivial $e^a{}_\mu = \delta^a{}_\mu$, so that, in the rest frame $u^\mu = (1, \mathbf{0})$

两种 $\ell = 1$ 矩都对应世界线有效场论中同一个耦合项 $-\int d\tau S^{ab}\Omega_{ab}$ 。为了和世界线匹配，我们选取共动框架为平凡框架 $e^a{}_\mu = \delta^a{}_\mu$ ，因此在静止系 $u^\mu = (1, \mathbf{0})$ 中

$$\frac{1}{2}\int d\tau S_{ab}\Omega^{ab} = \frac{1}{2}\int dx^0 S_\mu{}^\nu \overline{\Gamma}^\mu{}_{0\nu} \approx -\frac{1}{4m_{\rm Pl}}\int dx^0 S^{\mu\nu}\left(\partial_\mu \bar{h}_{0\nu} - \partial_\nu \bar{h}_{0\mu}\right),$$

(87)

so that

因此

$$\frac{1}{2}\int d\tau S_{ab}\Omega^{ab} \approx \frac{1}{4m_{\rm Pl}}\int dx^0 \left[2\left(\partial_0 S^{0i}\right)\bar{h}_{0i} + 2S^{0i}\partial_i \bar{h}_{00} - S^{ij}\left(\partial_i \bar{h}_{0j} - \partial_j \bar{h}_{0i}\right)\right].$$

(88)This matches the multipole expansion of $\overline{\tau}_{\mu\nu}$, provided that we identify

这与 $\overline{\tau}_{\mu\nu}$ 的多极展开一致，只要我们定义

$$S^{0i} = -P^0 X^i_{\rm CM}, S^{ij} = L^{ij}, \tag{89}$$

in which case $\partial_0 S^{0i} = -P^i$ vanishes in the rest frame. Therefore, in any coordinate system where $\mathbf{X}_{\rm CM} = 0$ (but $\mathbf{P}$ is not necessarily zero), the spin satisfies the ”covariant spin supplementary condition” $S_{ab}P^b = 0$.

此时 $\partial_0 S^{0i} = -P^i$ 在静止系中为零。因此，在任意满足 $\mathbf{X}_{\rm CM} = 0$ (但 $\mathbf{P}$ 不一定为零) 的坐标系中，自旋满足“协变自旋补充条件” $S_{ab}P^b = 0$ 。

Because the pseudotensor is conserved, none of the multipole terms considered so far can act as a source of gravitational radiation. Rather, they simply encode the (ADM) mass and angular momentum of the system as determined by measurements of the object’s gravitational field at spatial infinity. The radiative moments instead show up at second order in the multipole expansion and higher.

由于赝张量守恒，目前讨论的所有多极项都不能作为引力辐射的源，它们仅描述在空间无穷远测量引力场得到的系统 (ADM) 质量与角动量。辐射矩则出现在多极展开的二阶及更高阶。

The strategy for finding the relation between the moments in Eq. (57) and the weighted integrals $\int d^3\mathbf{x} x^{i_1}\cdots x^{i_\ell}\overline{\tau}^{\mu\nu}$ at order $\ell \geq 2$ follows along the same lines as above. In the rest frame, we decompose $\mu\nu$ into time and space components and project onto objects lying in irreducible representations of the permutation group. By the use of moment relations such as Eq. (81), and by removing traces, the resulting tensors are then projected onto irreducible tensors which correspond, in a general frame, to the irreducible representations of the $SO(3)$ little group of the composite object. For example, the averages $\int d^3\mathbf{x} x^{i_1}\cdots x^{i_\ell}\overline{\tau}^{\mu\nu}$ contain representations of dimensions $\leq 2\ell+1$ if $\mu\nu=00$, for $\mu\nu=0i$ we find representations with dimension $\leq 2\ell+3$, and for $\mu\nu=ij$, all dimensions $\leq 2\ell+5$ can in principle appear. By diffeomorphism invariance, these $SO(3)$ irreps couple linearly to the derivatives of the Riemann tensor, projected orthogonal to P^{μ} . (Couplings to the Ricci tensor also appear in the decomposition, but such "nonradiative" moments vanish on-shell and can be removed by field redefinitions of $\bar{g}_{\mu\nu}$ without any effect on physical observables.)

寻找式 (57) 中矩与 $\int d^3\mathbf{x}x^{i_1}\cdots x^{i_\ell}\tau^{\mu\nu}$ 在 $\ell\geq 2$ 阶加权积分之间关系的策略与上述思路一致。在静止系中，我们将 $\mu\nu$ 分解为时间分量和空间分量，并投影到置换群不可约表示对应的对象上。利用式 (81) 这类矩关系，并通过去除迹，最终得到的张量会被投影到不可约张量上；在一般坐标系中，这些不可约张量对应复合物体 $SO(3)$ 小群的不可约表示。例如，若 $\mu\nu=00$ ，平均量 $\int d^3\mathbf{x}x^{i_1}\cdots x^{i_\ell}\tau^{\mu\nu}$ 包含维数为 $\leq 2\ell+1$ 的表示；对于 $\mu\nu=0i$ ，我们得到维数为 $\leq 2\ell+3$ 的表示；对于 $\mu\nu=ij$ ，原则上所有维数 $\leq 2\ell+5$ 都可以出现。根据微分同胚不变性，这些 $SO(3)$ 不可约表示与投影到 P^{μ} 正交方向的黎曼张量导数线性耦合。(分解中也会出现与里奇张量的耦合，但这类“非辐射”矩在在壳条件下为零，可以通过重新定义 $\bar{g}_{\mu\nu}$ 场去掉，对物理可观测量没有任何影响。)

This procedure has been worked out in full generality [100]. One finds that an infinite number of coefficients from Eq. (54) contribute at each multipole order ℓ . For generic systems, each one of these contributions to $I^{a_1\cdots a_\ell}, J^{a_1\cdots a_\ell}$ is comparable magnitude, but if the sources are non-relativistic, this series of terms are suppressed by further powers of v and can be truncated at finite order. For example, for nonrelativistic systems, one finds that in the CM frame, the multipole expanded action takes the form

该过程已在完全通用的情况下推导完成 [100]。可以发现，在每个多极阶 ℓ ，式 (54) 都有无穷多个系数有贡献。对于一般系统，这些对 $I^{a_1\cdots a_\ell}, J^{a_1\cdots a_\ell}$ 的贡献每一项量级都相当，但如果源是非相对论性的，这一系列项会被更高次的 v 幂压低，可以截断到有限阶。例如，对于非相对论系统，质心系中多极展开作用量形式为

$$\Gamma^{(1)} \supset \int dx^0 \left[\frac{1}{2} I_{ij} E^{ij} - \frac{2}{3} J_{ij} B^{ij} + \frac{1}{6} I_{ijk} \nabla^i E^{ij} + \cdots\right], \tag{90}$$

where the electric quadrupole moment is (in $d=3$) [55]

其中电四极矩为 (在 $d=3$ 中)[55]

$$I^{ij} = \int d^3\mathbf{x} \left[\tau^{00} + \tau^{kk} - \frac{4}{3}\partial_0 \tau^{0k} x^k + \frac{11}{42}\partial_0^2 \tau^{00}\mathbf{x}^2 + \cdots\right] \left[x^i x^j\right]_{STF}, \tag{91}$$

whose non-trivial numerical coefficients reflect the underlying $SO(3)$ representation theory ("STF" denotes the operation of tensor symmetrization followed by the subtraction of traces with respect to the spatial metric in the comoving frame.). Similarly the magnetic quadrupole and electric octopole ($\ell=3$) are

其非平凡数值系数反映了 underlying $SO(3)$ 表示理论 (“STF”表示对张量做对称化，再减去随动系空间度量的迹的操作)。类似地，磁四极矩和电八极矩 ($\ell=3$) 为

$$J^{ij} = -\frac{1}{2}\int d^3\mathbf{x} \left[\varepsilon^{ikl}\tau^{0k} x^j x^l + \varepsilon^{jkl}\tau^{0k} x^i x^l + \cdots\right], \tag{92}$$

$$I^{ijk} = \int d^3\mathbf{x} \left[\tau^{00} + \tau^{ll} + \cdots\right] \left[x^i x^j\right]_{STF}, \tag{93}$$

respectively.

分别对应上述结果。

Equations (91)-(93) are sufficient to compute gravitational radiation to 1PN beyond the leading order quadrupole formula, given the components of $\overline{\tau}^{\mu\upsilon}$ to at least that accuracy. To organize the calculation, we work in a mixed momentum-time representation $\overline{\tau}^{\mu\upsilon}\left(x^0, \mathbf{k}\right) = \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \overline{\tau}^{\mu\upsilon}\left(x^0, \mathbf{x}\right)$, so that we can read off the moments by taking the soft graviton limit:

若已知 $\overline{\tau}^{\mu\upsilon}$ 的分量至少达到对应精度，式 (91)-(93) 就足以计算领头阶四极公式之外 1PN 阶的引力辐射。为了整理计算过程，我们在混合动量-时间表示 $\overline{\tau}^{\mu\upsilon}\left(x^0, \mathbf{k}\right) = \int d^3\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \overline{\tau}^{\mu\upsilon}\left(x^0, \mathbf{x}\right)$ 下工作，这样就可以通过取软引力子极限直接读出矩:

$$\overline{\tau}^{\mu\upsilon}\left(x^0, \mathbf{k} \to 0\right) = \sum_{\ell=0}^{\infty} \frac{(-i)^{\ell}}{\ell!} \left[\int d^3\mathbf{x} \overline{\tau}^{\mu\upsilon}\left(x^0, \mathbf{x}\right) x^{i_1} \cdots x^{i_\ell}\right] k_{i_1} \cdots k_{i_\ell}. \tag{94}$$

As an example, we determine the non-zero multipole moments of a compact binary system up to 1PN order in the velocity expansion. First, there are nonvanishing contributions to the electric moments at $\mathcal{O}\left(v^0\right)$, from the $\mu\upsilon = 00$ component

我们举一个例子，在速度展开中确定致密双星系统到 1PN 阶的非零多极矩。首先，$\mu\upsilon = 00$ 分量对 $\mathcal{O}\left(v^0\right)$ 阶电矩存在非零贡献

$$^{00} = \sum_A m_A \left[1 - i\mathbf{k}\cdot\mathbf{x}_A + \frac{1}{2!}\left(-i\mathbf{k}\cdot\mathbf{x}_A\right)^2 + + \frac{1}{3!}\left(-i\mathbf{k}\cdot\mathbf{x}_A\right)^3 + \cdots\right],$$

(95)

and therefore, in the CM frame, with $M = \sum_A m_A$

因此，在质心系中，当 $M = \sum_A m_A$ 时，

$$P^0 = \sum_A m_A + \mathcal{O}\left(v^2\right), \tag{96}$$

$$\mathbf{X}_{\mathrm{CM}} = \frac{1}{M} \sum_A m_A \mathbf{x}_A + \mathcal{O}\left(v^2\right), \tag{97}$$

$$I^{ij} = \sum_A m_A \left[x_A^i x_A^j\right]_{STF} + \mathcal{O}\left(v^2\right), \tag{98}$$

$$I^{ijk} = \sum m_A A \left[x_A^i x_A^j x_A^k\right]_{STF} + \mathcal{O}\left(v^2\right), \tag{99}$$

etc. As expected, the leading order multipole moments agree with the predictions of Newtonian theory. For example, the total mass of the system is just the sum $M = \sum_A m_A$ of the ADM masses of the binary constituents.

等等。不出所料，领头阶多极矩与牛顿理论的预测一致。例如，系统的总质量就是双星组分 ADM 质量之和 $M=\sum_A m_A$ 。

Similarly, the magnetic-type moments arise first at $\mathcal{O}\left(v^1\right)$, sourced by the $\mu v = \ 0i$ components of $\overline{\tau}^{\mu v}$

类似地，磁型矩首次出现在 $\mathcal{O}\left(v^1\right)$ 阶，由 $\overline{\tau}^{\mu v}$ 的 $\mu v = \ 0i$ 分量作为源产生

$$\overset{k}{\underset{\sim}{k}}\mathbf{v}^{i-1} = \sum_A m_A \mathbf{v}_A^i \Big[1 - i\mathbf{k}\cdot\mathbf{x}_A + \frac{1}{2!}(-i\mathbf{k}\cdot\mathbf{x}_A)^2 + \\ + \frac{1}{3!}(-i\mathbf{k}\cdot\mathbf{x}_A)^3 + \cdots\Big], \tag{100}$$

and therefore

以此类推

$$P^i = \sum_A m_A v_A^i + \mathcal{O}\left(v^3\right), \tag{101}$$

$$L^{ij} = \sum_A m_A \left(x_A^i v_A^j - x_A^j v_A^i\right) + \mathcal{O}\left(v^3\right), \tag{102}$$

$$J^{ij} = \frac{1}{2}\sum_A m_A \left[(\mathbf{x}_A \times \mathbf{v}_A)^i v_A^j + (i \leftrightarrow j)\right], \tag{103}$$

and so on at $\ell > 2$.

依此类推，在 $\ell > 2$ 阶也是如此。

At $\mathcal{O}\left(v^2\right), \overline{\tau}^{\mu v}$ is calculated from the sum of the Feynman diagrams in Fig. 5b-d. These depict both special relativistic corrections, as in Fig. 5b as well as the effects of gravitational interactions between the compact objects, Fig. 5c, d. To evaluate the diagrams, (c), (d), a complete table of integrals is provided by the standard one-loop integral

$\mathcal{O}\left(v^2\right), \overline{\tau}^{\mu v}$ 阶的结果由图 5b-d 中的费曼图求和计算得到。这些图既描绘了狭义相对论修正 (如图 5b 所示)，也描绘了致密天体之间引力相互作用的效应 (如图 5c、d 所示)。为计算图 (c) 和 (d)，标准单圈积分给出了完整的积分表

$$\int_{\mathbf{p}} \frac{1}{(\mathbf{p}^2)^\alpha\left[(\mathbf{p}+\mathbf{k})^2\right]^\beta} = \frac{1}{(4\pi)^{\frac{d-1}{2}}}\frac{\Gamma\left(\alpha+\beta-\frac{d-1}{2}\right)}{\Gamma(\alpha)\Gamma(\beta)} \\ \times \frac{\Gamma\left(\frac{d-1}{2}-\alpha\right)\Gamma\left(\frac{d-1}{2}-\beta\right)}{\Gamma(d-1-\alpha-\beta)}\left(\mathbf{k}^2\right)^{\frac{d-1}{2}-\alpha-\beta}, \tag{104}$$

and the Fourier transform

以及傅里叶变换

$$\int_{\mathbf{p}} \frac{e^{i\mathbf{p}\cdot\mathbf{x}}}{(\mathbf{p}^2)^{\alpha}} = \frac{1}{(4\pi)^{\frac{d-1}{2}}} \frac{\Gamma\left(\frac{d-1}{2}-\alpha\right)}{\Gamma(\alpha)} \left(\frac{\mathbf{x}^2}{4}\right)^{\alpha-\frac{d-1}{2}}. \tag{105}$$

One finds that including effects up to 1PN order, the electric-type $\ell < 2$ moments are

可以发现，计入直到 1PN 阶的效应后，电型 $\ell < 2$ 矩为

$$P^0 = \sum_A \bar{m}_A + \mathcal{O}\left(v^4\right) \tag{106}$$

$$\mathbf{X}_{\mathrm{CM}} = \frac{1}{M} \sum_A \bar{m}_A \mathbf{x}_A + \mathcal{O}\left(v^4\right), \tag{107}$$

which are averages weighted over a ”renormalized” particle mass defined as

它们是按“重整化”粒子质量定义的加权平均，该质量定义为

$$\bar{m}_A = m_A \left[1 + \frac{1}{2}\mathbf{v}_A^2 - \frac{1}{2}\sum_B \frac{G_N m_B}{|\mathbf{x}_A - \mathbf{x}_B|}\right] \tag{108}$$

(We do not quote here the order v^2 corrections to magnetic moments with $\ell < 2$ since these are not necessary to predict the 1PN accurate observables in the CM frame. Note that, in any case, given $P^0, \mathbf{X}_{\mathrm{CM}}$ to 1PN order, the spatial momentum can be obtained from the moment relation $\mathbf{P}^i/P^0 = d\mathbf{X}^i_{\mathrm{CM}}/dx^0$ without the need to explicitly calculate the 1PN corrections to $\overline{\tau}^{0i}$.) The relevant $\ell \geq 2$ moments at 1PN are then I^{ijk} in Eq. (99), J^{ij} in Eq. (103), and the electric quadrupole moment to $\mathcal{O}\left(v^2\right)$, obtained by inserting $\overline{\tau}^{\mu\nu}$ into Eq. (91)

(我们在此不引述带 $\ell < 2$ 的磁矩的 v^2 阶修正，因为这些修正对于预测质心系中 1PN 精度的可观测量而言并非必要。注意, 无论如何, 给定到 1PN 阶的 $P^0, \mathbf{X}_{\mathrm{CM}}$, 都可以通过矩关系 $\mathbf{P}^i/P^0 = d\mathbf{X}^i_{\mathrm{CM}}/dx^0$ 得到空间动量，无需显式计算对 $\overline{\tau}^{0i}$ 的 1PN 修正。) 此后，1PN 阶的相关 $\ell \geq 2$ 矩就是式 (99) 中的 I^{ijk} 、式 (103) 中的 J^{ij} ，以及到 $\mathcal{O}\left(v^2\right)$ 阶的电四极矩，后者通过将 $\overline{\tau}^{\mu\nu}$ 代入式 (91) 得到

$$I^{ij} = \sum_A m_A \left(1 + \frac{3}{2}\mathbf{v}_A^2 - \sum_B \frac{G_N m_B}{|\mathbf{x}_A - \mathbf{x}_B|} + \frac{11}{42}\frac{d^2}{dt^2}\mathbf{x}_A^2 - \frac{4}{3}\frac{d}{dt}\mathbf{x}_A \cdot \mathbf{v}_A\right)\left[x_A^i x_A^j\right]_{STF},$$

(109)

where the time derivatives act on anything to their right.

其中时间导数作用于其右侧的所有项。

Having obtained explicit formulae for the multipole moments in terms of the microscopic degrees of freedom $\mathbf{x}_A$, it is now possible to predict the radiative corrections to binary dynamics. For example, the time averaged power into gravitons in the $\ell = 2$ partial wave can be calculated by inserting the expression for I_{ij} into Eq. (65) and summing over final state graviton polarizations/integrating over phase space (useful formulas for performing polarization sums and angular integrals can be found, e.g., in [115]), leading to the classic formula

在得到以微观自由度 $\mathbf{x}_A$ 表示的多极矩显式公式后，现在可以预测双星动力学的辐射修正。例如，可以通过将 I_{ij} 的表达式代入式 (65)，并对末态引力子极化求和/对相空间积分 (例如，极化求和和角积分的有用公式可参见文献 [115])，计算得到 $\ell = 2$ 分波中引力子的时间平均辐射功率，由此得到经典公式

$$P_{GW}^{\ell=2} = \frac{G_N}{5}\left\langle \frac{d^3 I_{ij}}{dt^3}\frac{d^3 I_{ij}}{dt^3}\right\rangle, \tag{110}$$

while the leading PN waveform at $\mathcal{I}^+$ follows from Eq. (37) and Eq. (63)

而 $\mathcal{I}^+$ 阶的领头后牛顿波形可由式 (37) 和式 (63) 得到

$$h_{ij}(u) = \frac{4G_N}{|\mathbf{x}|}\frac{d^2}{du^2}I_{ij}(u), \tag{111}$$

where I_{ij} is evaluated on-shell, i.e., on the solution of the PN equations of motion for the orbits $\mathbf{x}_A$. These equations of motion include ”conservative” terms from integrating out potential modes, as discussed in section ”Two-Body Potentials,” as well as radiative corrections due to the backreaction of the emitted gravitational waves on the particle trajectories. The latter can be calculated directly in ZSO IR as we explain in the next section.

其中 I_{ij} 是在壳求值的，即在轨道 $\mathbf{x}_A$ 的后牛顿运动方程的解上求值。这些运动方程既包含了积去势模得到的“保守”项 (如“两体势”一节所述)，也包含了所发射引力波对粒子轨道反作用带来的辐射修正。我们将在下一节说明，后者可以直接在 ZSO 红外方案中计算。

Time Non-locality: Radiation Reaction, Black Hole Event Horizons

时间非定域性: 辐射反作用、黑洞事件视界

In this section we provide a brief overview of two applications of the worldline EFT Eq. (57) to dissipative processes in binary dynamics. In section ”Radiation Reaction in Zee IR, we discuss radiation damping of the particle orbits at leading PN order, following [46]. Even though the effects of radiation reaction are local in time at the level of the equations of motion (at least at leading order), they cannot be obtained by variation of a local Lagrangian. It is essential to employ Schwinger-Keldysh contours in the path integral in order to ensure causal time evolution.

本节我们简要介绍世界线有效场论 (EFT) 式 (57) 在双星动力学耗散过程中的两个应用。在“Zee 红外中的辐射反作用”一节中，我们遵循文献 [46] 讨论主导 PN 阶下粒子轨道的辐射阻尼。尽管在运动方程层面，辐射反作用效应是时间定域的 (至少在领头阶如此)，却无法通过对定域拉格朗日量变分得到。为保证因果时间演化，必须在路径积分中使用施温格-凯尔迪什 contour。

In section ”Event Horizon Dynamics in TSOU,” we incorporate the effects of black hole horizon dynamics. The absorption of energy and momentum by the horizon implies the existence of massless degrees of freedom on the worldline EFT, which are described by UV version of Eq. (57). When integrated out, these modes generate a nonlocal in time effective action for the particle trajectories, but local (and time reversal odd) friction forces at the level of the equations of motion.

在“TSOU 中的事件视界动力学”一节中，我们纳入黑洞视界动力学的效应。视界对能量动量的吸收意味着世界线 EFT 中存在无质量自由度，这些自由度由式 (57) 的紫外版本描述。积分掉这些自由度后，会为粒子轨迹生成一个时间非定域的有效作用量，但在运动方程层面得到定域 (且时间反演奇) 的摩擦力。

Radiation Reaction in The IR

红外区的辐射反作用

Ultimately, the goal of the PN expansion is to calculate both the gravitational waveform as seen by observers at infinity. At sufficiently high orders in velocity, this involves determining how the particle orbits $\mathbf{x}_A(t)$ are affected by the emission of radiation. Such radiation reaction effects are encoded in the in-in effective action $\Gamma[x_A, \bar{g}; \tilde{x}_A, \tilde{g}]$ in Eq. (32) and the resulting equations of motion Eq. (34). Because the in-in action is generated by integrating out radiation, it is in general nonlocal in time.

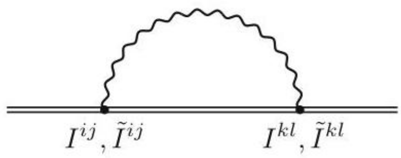
PN 展开的最终目标是计算无穷远观测者观测到的引力波形。在足够高的速度阶下，计算需要确定粒子轨道 $\mathbf{x}_A(t)$ 受辐射发射的影响。这类辐射反作用效应被编码在式 (32) 的入-出有效作用量 $\Gamma[x_A, \bar{g}; \tilde{x}_A, \tilde{g}]$ 和由此得到的运动方程式 (34) 中。由于入-出作用量是通过积掉辐射得到的，它一般是时间非定域的。

Since this non-locality in time occurs over scales that are of the same size as the wavelength of the radiation wavelength, it is practical to compute $\Gamma[x_A, \bar{g}, \tilde{x}_A; \tilde{\bar{g}}]$ by first constructing the worldline theory ZSO IR and then calculating vacuum Feynman graphs in that theory, as first advocated in Ref. [46]. To illustrate this procedure, we consider the effects of radiation reaction on a composite system described by the action in Eq. (57). In that theory, integrating out radiation generates a correction $\Delta\Gamma[x_A\bar{g}; \tilde{x}_A, \tilde{\bar{g}}]$ to the in-in effective action which is in general nonlocal in time. For example, the self-energy diagram which represents a sum over four independent in-in Wick contractions, gives the leading order radiation reaction effect due to quadrupole emission

由于这种时间非定域性发生在与辐射波长相同的尺度上，按照文献 [46] 首先提出的方案，先构造世界线理论 ZSO 红外区，再在该理论中计算真空费曼图是计算 $\Gamma[x_A, \bar{g}, \tilde{x}_A; \tilde{\bar{g}}]$ 的实用方法。为说明这一步骤，我们考虑辐射反作用对式 (57) 作用量描述的复合系统的影响。在该理论中，积掉辐射会对入-出有效作用量产生一个修正项 $\Delta\Gamma[x_A\bar{g}; \tilde{x}_A, \tilde{\bar{g}}]$，该修正项一般是时间非定域的。例如，代表对四个独立入-出维克收缩求和的自能图，给出了四极辐射 leading order 的辐射反作用效应

$$
\begin{aligned}
i\Delta\Gamma[x_A, \bar{g}; \tilde{x}_A, \tilde{\bar{g}}] &= \frac{1}{2!}\int dt_1 dt_2 \left(\frac{i}{2} I^{ij}(t_1)\right)\langle E_{ij}(t_1, 0) E_{kl}(t_2, 0)\rangle \left(\frac{i}{2} I^{kl}(t_2)\right) \\
&+ \int dt_1 dt_2 \left(\frac{i}{2} I^{ij}(t_1)\right)\langle E_{ij}(t_1, 0) \widetilde{E}_{kl}(t_2, 0)\rangle \left(-\frac{i}{2} \tilde{I}^{kl}(t_2)\right) \\
&+ \frac{1}{2!}\int dt_1 dt_2 \left(-\frac{i}{2} \tilde{I}^{ij}(t_1)\right)\langle \widetilde{E}_{ij}(t_1, 0) \widetilde{E}_{kl}(t_2, 0)\rangle \left(-\frac{i}{2} \tilde{I}^{kl}(t_2)\right),
\end{aligned}
$$

(113)

$i\Delta\Gamma[x_A, \bar{g}; \tilde{x}_A, \tilde{\bar{g}}] =$  $\sim Lv^5,$

(112)

(recall from section "Perturbative Binary Dynamics" that $\left\langle E_{ij}\left(t_1,0\right)E_{kl}\left(t_2,0\right)\right\rangle$ is the time-ordered expectation value while $\left\langle E_{ij}\left(t_1,0\right)\widetilde{E}_{kl}\left(t_2,0\right)\right\rangle = \langle 0 \mid E_{kl}\left(t_2,0\right)\ \ E_{ij}\left(t_1,0\right)|0\rangle$ is a Wightman two-point function) and $\left\langle \widetilde{E}_{ij}\left(t_1,0\right)\widetilde{E}_{kl}\left(t_2,0\right)\right\rangle$ is anti-time-ordered (Dyson)). The equation of motion for each particle $\mathbf{x}_A$ is the extremum of $\Gamma\left[x_A,\bar{g};\tilde{x}_A,\tilde{\bar{g}}\right]$, after setting $\tilde{x}_A = x_A$ and setting to zero the background field, $\bar{g}_{\mu\nu} = \tilde{\bar{g}}_{\mu\nu} = \eta_{\mu\nu}$. In particular, the correction $\Delta\Gamma\left[x_A,\bar{g};\tilde{x}_A,\tilde{\bar{g}}\right]$ calculated above contributes to the radiation damping force on particle A

(回顾“微扰双星动力学”一节可知，$\left\langle E_{ij}\left(t_1,0\right)E_{kl}\left(t_2,0\right)\right\rangle$ 是时间有序期望值，$\left\langle E_{ij}\left(t_1,0\right)\widetilde{E}_{kl}\left(t_2,0\right)\right\rangle = \langle 0 \mid E_{kl}\left(t_2,0\right)\ \ E_{ij}\left(t_1,0\right)|0\rangle$ 是维特特曼两点函数，$\left\langle \widetilde{E}_{ij}\left(t_1,0\right)\widetilde{E}_{kl}\left(t_2,0\right)\right\rangle$ 是反时间有序 (戴森) 两点函数)。每个粒子 $\mathbf{x}_A$ 的运动方程是 $\Gamma\left[x_A,\bar{g};\tilde{x}_A,\tilde{\bar{g}}\right]$ 的极值，固定 $\tilde{x}_A = x_A$ 并令背景场 $\bar{g}_{\mu\nu} = \tilde{\bar{g}}_{\mu\nu} = \eta_{\mu\nu}$ 为零后即可得到。特别地，上文计算得到的修正项 $\Delta\Gamma\left[x_A,\bar{g};\tilde{x}_A,\tilde{\bar{g}}\right]$ 对粒子 A 的辐射阻尼力有贡献

$$\mathbf{F}_A(t) = \left.\frac{\delta}{\delta\mathbf{x}_A(t)}\Delta\Gamma\left[x_A,\eta;\tilde{x}_A,\eta\right]\right|_{\tilde{\mathbf{x}}_A=\mathbf{x}_A}$$

$$= \frac{i}{4}\int dt_1 dt_2\left(\frac{\delta I^{ij}\left(t_1\right)}{\delta\mathbf{x}_A(t)}\right)\left[\left\langle E_{ij}\left(t_1,0\right)E_{kl}\left(t_2,0\right)\right\rangle - \left\langle E_{ij}\left(t_1,0\right)\widetilde{E}_{kl}\left(t_2,0\right)\right\rangle\right]I^{kl}\left(t_2\right).$$

(114)

It is built into the in-in formalism that the action is a real quantity and that the equations of motion are causal. In particular, the expression $\left\langle E_{ij}\left(t_1,0\right)E_{kl}\left(t_2,0\right)\right\rangle - \left\langle E_{ij}\left(t_1,0\right)\widetilde{E}_{kl}\left(t_2,0\right)\right\rangle$ is proportional to the retarded two-point Green’s function of the electric curvature operator, as a consequence of the general operator identity:

入-出形式体系本身就要求作用量是实量，且运动方程满足因果性。根据一般算符恒等式，表达式 $\left\langle E_{ij}\left(t_1,0\right)E_{kl}\left(t_2,0\right)\right\rangle - \left\langle E_{ij}\left(t_1,0\right)\widetilde{E}_{kl}\left(t_2,0\right)\right\rangle$ 正比于电曲率算符的推迟两点格林函数:

$$T\left[\mathcal{O}_1\left(t_1\right)\mathcal{O}\left(t_2\right)\right] - \mathcal{O}_1\left(t_1\right)\mathcal{O}_2\left(t_2\right) = \theta\left(t_1 - t_2\right)\left[\mathcal{O}_1\left(t_1\right), \mathcal{O}_1\left(t_1\right)\right]. \tag{115}$$

Inserting the linearized Riemann tensor into the correlator

将线性化黎曼张量代入关联函数

$$E_{ij} = R_{0i0j} \approx -\frac{1}{2}\partial_0^2\bar{h}_{ij} + \frac{1}{2}\partial_0\partial_i\bar{h}_{0j} + \frac{1}{2}\partial_0\partial_j\bar{h}_{0i} - \frac{1}{2}\partial_i\partial_j\bar{h}_{00}, \tag{116}$$

and using rotational symmetry to reduce the angular integrals, we have

再利用旋转对称性约化角积分，我们得到

$$\langle 0 \left| \left[E_{ij}(t_1,0), E_{kl}(t_2,0) \right] \right| 0 \rangle = \int \frac{d^3\mathbf{q}}{(2\pi)^3 2|\mathbf{q}|} \frac{|\mathbf{q}|^4}{20m_{\mathrm{Pl}}^2} \left[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl} \right]$$

$$\times \left(e^{-i|\mathbf{q}|(t_1-t_2)} - e^{i|\mathbf{q}|(t_1-t_2)} \right) \tag{117}$$

$$= \frac{4G_N}{5} \left[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl} \right] \left(i\frac{d}{dt_1} \right)^5 \delta(t_1 - t_2).$$

Because of the step function $\theta(t_1 - t_2)$ in Eq. (115), encoding causality, integrating out the radiation results in a nonlocal effective action. However, at this PN order, the equations of motion are still local

由于式 (115) 中编码因果性的阶跃函数 $\theta(t_1 - t_2)$，积去辐射后会得到一个非局域有效作用量。但在该后牛顿阶下，运动方程仍然是局域的

$$\mathbf{F}_A^i(t) = \frac{\delta}{\delta \mathbf{x}_A^i(t)} \Delta\Gamma[x_A, \tilde{x}_A] \bigg|_{\tilde{\mathbf{x}}_A = \mathbf{x}_A} = -\frac{2G_N}{5} m_A \mathbf{x}_A^j \frac{d^5}{dt^5} I^{ij}(t), \tag{118}$$

as well as odd under time reversal, as is characteristic of dissipation. By employing Schwinger-Keldysh boundary conditions in the EFT, we have reproduced [46], the 2.5PN Burke-Thorne radiation reaction force, derived originally by purely classical methods [16]. The instantaneous loss of mechanical energy and angular momentum into radiation are then

且在时间反演下是奇的，这正是耗散的特征。通过在有效场论中采用施温格-凯尔迪什边界条件，我们重现了文献 [46] 的结果，即原本由纯经典方法推导得到的 2.5PN 伯克-索恩辐射反作用力 [16]。机械能和角动量向辐射的瞬时损失为

$$\frac{d}{dt} E = \sum_A \mathbf{v}_A \cdot \mathbf{F}_A = -\frac{G_N}{5} \frac{dI^{ij}(t)}{dt} \frac{d^5 I^{ij}(t)}{dt^5}, \tag{119}$$

$$\frac{d}{dt} J^i = \sum (\mathbf{x}_A \times \mathbf{F}_A)^i = -\frac{2G_N}{5} \varepsilon_{ijk} I^{jl}(t) \frac{d^5 I^{kl}(t)}{dt^5}. \tag{120}$$

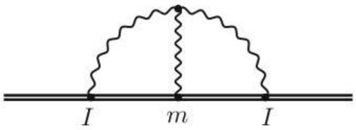
Note in particular the time average over many orbital cycles agrees with the standard quadrupole radiation formula in Eq. (110).

尤其要注意，对多个轨道周期做时间平均后，结果与式 (110) 中的标准四极辐射公式一致

The procedure outlined in this example is completely systematic and has been extended to higher orders in the PN expansion. For example, at 4PN order, there is a tail correction to the in-in action, from diagrams such as

本例中概述的流程是完全系统化的，现已被推广到后牛顿展开的更高阶。例如，在 4PN 阶，入作用存在一个尾巴修正，来自如下这类图

(121)

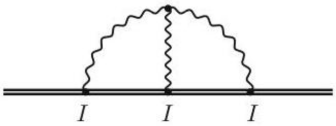


$$\sim Lv^8$$

while at 5PN, there are nonlinear "hereditary" or "memory effects" in which the equations of motion at a given time depend on the quadrupole moment of the system at earlier times, such as the term

而在 5PN 阶，存在非线性“遗传效应”即“记忆效应”: 某一时刻的运动方程依赖于系统在更早时刻的四极矩，例如该项

(122)



$$\sim Lv^{10}.$$

In these calculations, a combination of field theoretic techniques (EFTs, dimensional regularization, the Schwinger-Keldysh path integral, etc.) plays an essential role in obtaining self-consistent, ambiguity-free results for dynamical observables. A review of these intricate higher-order effects is beyond the scope of this chapter, and the reader is referred to the original literature, compiled in Ref. [61], for the technical details.

在这些计算中，场论技术 (有效场论、维度正规化、施温格-凯尔迪什路径积分等) 的结合，对得到自洽、无歧义的动力学观测量结果起到了核心作用。对这些复杂的高阶效应的综述超出了本章的范围，技术细节请读者查阅文献 [61] 汇编的原始文献

Event Horizon Dynamics in ZSOUV

ZSOUV 中的事件视界动力学

The formalism as described so far is adequate for compact objects whose internal structure is gapped, so that gravitational interactions at scales longer than the orbital radius cannot irreversibly modify the intrinsic properties of the object. For black holes, though, this frequency gap is of order $1/r_s$, so that finite size effects will come in at some finite order in $r_s/r \sim v^2$ in the PN expansion. In order to have a fully systematic treatment of PN black hole binary dynamics, such dissipative effects cannot be neglected.

到目前为止所描述的形式体系适用于内部结构存在能隙的致密天体, 因此轨道半径以上尺度的引力相互作用不会不可逆地改变天体的固有属性。但对于黑洞而言，该频率缺口的量级为 $1/r_s$，因此在 PN 展开中有限尺寸效应会在 $r_s/r \sim v^2$ 的某有限阶显现。要对 PN 黑洞双星动力学进行完全系统的处理，这类耗散效应是不能忽略的。

For instance, from black hole perturbation theory [102], it is known that the change in mass due to tidal heating induced by a small binary companion in a bound orbit appears at order 4PN [92] in the Schwarzschild case and becomes enhanced to 2.5PN [108] for (near extremal) Kerr black holes. In the latter case, the tidal interactions can actually decrease [28, 93] the mass of the black hole as a consequence of stimulated emission (”rotational superradiance”) [107, 118], a field theoretic realization of the Penrose process [91] of energy extraction from the black hole’s ergosphere.

例如，根据黑洞微扰理论 [102]，我们已知在束缚轨道上，小型伴星诱导的潮汐加热引起的质量变化，在史瓦西情形下出现在 4PN 阶 [92]，而对于 (近极端) 克尔黑洞则会提升至 2.5PN 阶 [108]。在后一种情形下，潮汐作用实际上可以降低 [28, 93] 黑洞质量，这是受激辐射 (“转动超辐射”)[107, 118] 的结果，而该过程是从黑洞能层提取能量的彭罗斯过程 [91] 在量子场论中的实现。

On general grounds [17], dissipation, e.g., flux of energy and angular momentum across the surface of the compact star, signals the presence of a continuum spectrum of localized degrees of freedom that couple to gravity in the bulk spacetime. For a neutron star, the additional degrees of freedom correspond to the low-lying hydrodynamic modes of nuclear matter, while for classical black holes, the horizon fluctuations are presumably related to the quasinormal mode solutions of the Teukolsky equation. Regardless of the microscopic origin of the internal degrees of freedom, their presence has an effect on the binary inspiral dynamics at some order in the PN expansion.

一般而言 [17]，耗散 (即能量和角动量穿过致密恒星表面的通量) 表明存在局域化自由度的连续谱，这些自由度与背景体时空的引力耦合。对于中子星，额外自由度对应核物质的低能流体力学模式；而对于经典黑洞，视界涨落推测与泰奥科夫斯基方程的准正则模解相关。无论内自由度的微观起源是什么，它们的存在都会在 PN 展开的某一阶对双星并合动力学产生影响。

It is therefore useful to have a way of incorporating the effects of dissipation directly in the worldline description of the compact objects, without explicitly having to track the evolution of the internal modes themselves. An EFT framework for this was first introduced in Ref. [52], which describes the long wavelength dissipative response of compact objects to external gravitational perturbations, by ”integrating in” a quantum mechanical $0+1$ -dimensional defect field theory of degrees of freedom localized on the worldline.

因此，有一种方法可以直接将耗散效应纳入致密天体的世界线描述，而无需显式追踪内部模式本身的演化，这是十分实用的。该 EFT 框架最早由文献 [52] 提出，它通过“积入”一个位于世界线上的量子力学 $0+1$ 维缺陷场论 (其自由度定域在世界线上)，来描述致密天体对外部引力微扰的长波耗散响应。

Independent of their UV origin, in the long-distance limit, these modes have local diff and reparameterization invariant couplings to the spacetime curvature. Organizing the algebra of defect operators in terms of the linearly realized rotations about the objects spatial location, the symmetries of the EFT guarantee that the

无论这些自由度的紫外起源如何，在长程极限下，它们都具有局域微分不变和重参数化不变的耦合，与时空曲率耦合。我们围绕天体所在空间位置，按线性实现的转动整理缺陷算符代数后，EFT 的对称性保证

Lagrangian must be of identical form to Eq. (57), where now the multipole moments $I_{a_1\cdots a_\ell}, J_{a_1\cdots a_\ell}$, should be regarded as a set of composite operators constructed out of the microscopic degrees of freedom, acting on some internal Hilbert space of physical states. As long as we probe the system with slowly varying fields, the compact object itself also appears as a Zoomed Out Single Object, even if its internal structure is arbitrarily complicated.

拉格朗日量的形式必须与式 (57) 完全相同，此时多极矩 $I_{a_1 \cdots a_\ell}, J_{a_1 \cdots a_\ell}$ 应被视作由微观自由度构造出的一组复合算符，作用于物理态的某个内希尔伯特空间。只要我们用缓变场探测该系统，即便致密天体的内部结构复杂程度任意，它本身也会表现为一个放大化单物体。

We do not need to know what the internal modes are in order to make predictions in the infrared. In this case, long-distance observables can be calculated in terms of the correlation functions of the multipole operators, which in turn are determined by a matching calculation to the UV theory. The power counting of the EFT indicates that at long distances, the leading contribution is from the two-point correlators of the electric and magnetic quadrupole operators

我们无需知道内部模式具体是什么，就能在红外区做出预言。在这套框架下，长程可观测量可以用多极算符的关联函数计算，而关联函数本身可以通过匹配紫外理论得到。EFT 的幂计数表明，在长距离下，领头贡献来自电四极和磁四极算符的两点关联函数

$$\langle I_{ab}(\tau) I_{cd}(0) \rangle, \langle J_{ab}(\tau) J_{cd}(0) \rangle$$

evaluated in the equilibrium (pure or mixed) state of the object. Predictions in the EFT, in powers of $\omega \mathcal{R} \ll 1$, are systematically improvable by including more multipoles, higher-point correlators, or perturbative graviton interactions which scale as powers of $G_N M \omega \lesssim \omega \mathcal{R} \ll 1$.

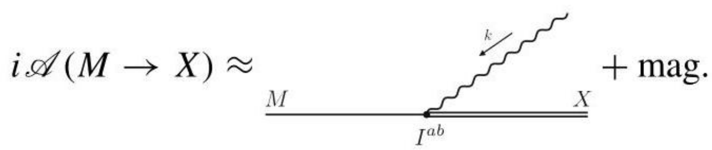
在天体的平衡 (纯态或混合态) 下计算得到。有效场论中按 $\omega \mathcal{R} \ll 1$ 幂次展开的预言可通过纳入更多多极、高阶关联函数，或按 $G_N M \omega \lesssim \omega \mathcal{R} \ll 1$ 幂次标度的微扰引力子相互作用系统地改进。

To match to this ZSOUV, we compute on-shell graviton scattering off an isolated object in the EFT, using Eq. (57), and compare it to the low-frequency limit of the corresponding observable in the full theory. As an example, consider graviton absorption by a Schwarzschild black hole, which in the EFT has the matrix element

为匹配该 ZSOUV，我们利用式 (57) 计算 EFT 中引力子在孤立天体上的 on-shell 散射，并将其与完整理论中对应可观测量的低频极限比较。举个例子，考虑史瓦西黑洞对引力子的吸收，其在 EFT 中的矩阵元为

$$= \frac{i}{2m_{Pl}} \int dt e^{-i\omega t} \langle X | I_{ab}(t) | M \rangle \times \langle 0 | E_{ab}(t, 0) | k, h \rangle + \text{ mag.},$$

(123)



in the rest frame, to leading order in $1/m_{Pl}$. Here, the matrix element $\langle 0 | E_{ab}(t, 0) | k, h \rangle$ between the one-graviton state of four-momentum $k^\mu \left(k^0 = \omega > 0\right)$, helicity $h = \pm 2$, and the vacuum is readily computed by standard canonical quantization of the Einstein-Hilbert Lagrangian [36], as established in [30 − 32, 39, 64] . The transition matrix element $\langle X | I_{ab}(t) | M \rangle$ from the initial black hole of mass M to some unknown final state $|X\rangle$ is not calculable in the EFT, but assuming unitarity

> 在静止系中，该结果是 $1/m_{Pl}$ 阶的领头项。此处，四动量为 $k^\mu\left(k^0 = \omega > 0\right)$、螺旋度为 $h = \pm 2$ 的单引力子态与真空之间的矩阵元 $\langle 0 | E_{ab}(t,0) | k, h \rangle$ 可通过爱因斯坦-希尔伯特拉格朗日量的标准正则量子化直接计算，相关结论已记录在 [30 − 32, 39, 64] 中。初始质量为 M 的黑洞到任意未知末态 $|X\rangle$ 的跃迁矩阵元 $\langle X | I_{ab}(t) | M \rangle$ 无法在有效场论框架下计算，但利用幺正性假设

$$\sum_X |X\rangle\langle X| = \mathbf{1}_{\mathcal{H}} \tag{124}$$

we can express the inclusive absorption cross section for a graviton incident on the horizon

> 我们可以将入射到视界的引力子的 inclusive 吸收截面

$$\sigma_{abs}(\omega) = \lim_{T\to\infty} \frac{1}{2\omega} \sum_X \frac{|\mathcal{A}(M \to X)|^2}{T} \tag{125}$$

$$= G_N \pi \omega^3 \int dt e^{i\omega t} \varepsilon^*_{cd,h}(k) \langle I_{cd}(t) I_{ab}(0) \rangle \varepsilon_{ab,h}(k) + \text{ mag.},$$

in terms of the two-point correlators of the $\ell = 2$ multipole operators in the initial state of a black hole of mass M and spin J .

> 表示为质量 M 、自旋 J 的黑洞初态中，$\ell = 2$ 多极算符两点关联函数的函数。

The cross section $\sigma_{abs}(\omega)$ is a physical quantity that can be compared against the predictions of classical general relativity in the limit $r_s\omega \ll 1$, where the EFT description is useful. Using the classical absorption probabilities calculated in Refs. [87,107], one finds that $\sigma_{abs}(\omega) \approx 4\pi r_s^6 \omega^4/45$, and exploiting the rotational invariance of the Schwarzschild black hole to write

> 截面 $\sigma_{abs}(\omega)$ 是物理量，可以在有效场论描述适用的 $r_s\omega \ll 1$ 极限下与经典广义相对论的预言对比。利用文献 [87,107] 中计算得到的经典吸收概率，可得 $\sigma_{abs}(\omega) \approx 4\pi r_s^6 \omega^4/45$ ，再利用史瓦西黑洞的旋转不变性写出

$$\int dt e^{i\omega t} \langle I_{ab}(t) I_{cd}(0) \rangle = \frac{1}{2}\left[\eta^\perp_{ac}\eta^\perp_{bd} + \eta^\perp_{ad}\eta^\perp_{bc} - \frac{2}{3}\eta^\perp_{ab}\eta^\perp_{cd}\right] A^E_+(\omega), \tag{126}$$

($\eta^\perp_{ab} = \eta_{ab} - p_a p_b/M^2$ is the spatial metric in the black hole’s rest frame), one finds that the frequency space correlators are to lowest order

> (其中 $\eta^\perp_{ab} = \eta_{ab} - p_a p_b/M^2$ 是黑洞静止系中的空间度规)，可以发现频率空间关联函数的最低阶形式为

$$A^E_+(\omega) = A^B_+(\omega) \approx 2\theta(\omega) r_s^6 \omega / 45 G_N, \tag{127}$$

where the factor $\theta(\omega)$ ensures that for a classical black hole, the graviton emission matrix element BH $\rightarrow$ BH$'+$ graviton is zero.

式中因子 $\theta(\omega)$ 保证经典黑洞的引力子发射矩阵元 BH $\rightarrow$ BH$'+$ 引力子为零。

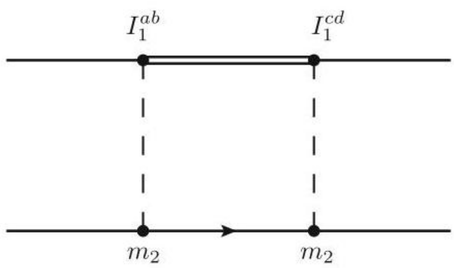
The point of this exercise is that the same correlators that one extracts from on-shell observables in the one-body sector also control off-shell graviton exchange processes in the two-body sector, where the binary dynamics is described by NRGR. By including diagrams with insertions of the multipole operators I_{ab}, J_{ab} , it becomes possible to include the effects of horizon dynamics in the EFT while retaining a worldline description of the binary constituents.

该推导的意义在于，单体部分从 on-shell 观测量提取得到的关联函数，同样控制着双体部分的 off-shell 引力子交换过程，而双体部分的双星动力学由非相对论广义相对论 (NRGR) 描述。通过引入多极算符 I_{ab}, J_{ab} 的插入图，就可以在有效场论中计入视界动力学的效应，同时保留对双星组分的世界线描述。

For example, single graviton exchange between two black holes generates a tidal friction (T -odd) term in the two-body equations of motion associated with the excitation of horizon modes, leading to a flux of energy across the event horizon. This is encoded in the box diagram contribution to the in-in effective action of the binary trajectories in the black hole binary, associated with the absorption of gravitational energy by the horizons, follow from variation of the effective action, $\mathbf{F}_1 = -\mathbf{F}_2 = \delta\Delta\Gamma/\delta\mathbf{x}_1|_{\tilde{x}_A = x_A}$.

例如，两个黑洞之间的单引力子交换会在双体运动方程中产生与视界模式激发相关的潮汐摩擦 (T 奇) 项，导致能量流过事件视界。这一效应编码在黑洞双星轨迹的 in-in 有效作用量的方框图贡献中，对应视界对引力能的吸收，可通过对有效作用量变分得到，$\mathbf{F}_1 = -\mathbf{F}_2 = \delta\Delta\Gamma/\delta\mathbf{x}_1|_{\tilde{x}_A = x_A}$.

$$i\Delta\Gamma[x_A, \eta; \tilde{x}_A, \eta] \supset$$



$$+ (1 \leftrightarrow 2),$$

(128)

plus a similar magnetic contribution. Given the form of the correlators $\langle I^{ab} I^{cd} \rangle$, $\langle J^{ab} J^{cd} \rangle$, this is a non-local in time quantity. The frictional force on the particle

加上类似的磁贡献。给定关联函数 $\langle I^{ab} I^{cd} \rangle$、$\langle J^{ab} J^{cd} \rangle$ 的形式后，该作用量是时间非定域量。粒子受到的摩擦力

The variation of Eq. (128), with respect to the particle trajectories, is causal, in the sense that it depends only the retarded Green's function

对式 (128) 关于粒子轨迹变分的结果是因果的，因为它仅依赖推迟格林函数

$$G_R^{ab,cd}(\tau) = -i\theta(\tau)\left\langle\left[I^{ab}(\tau), I^{cd}(0)\right]\right\rangle, \tag{129}$$

of the multipole operators on the black hole worldline. This again is a consequence of the in-in Feynman rules, together with the operator identity Eq. (115). Note that the retarded Green's function is not technically the same as the Wightman function in Eq. (126). They are, however, related by a dispersion relation, which in frequency space reads

黑洞世界线上多极算符的推迟格林函数。这再次是 in-in 费曼规则与式 (115) 算符恒等式共同作用的结果。注意严格来说，推迟格林函数与式 (126) 中的威曼函数并不相同，但二者满足色散关系，在频率空间中该关系可写为

$$G^R(\omega) = i\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{A_+(\omega') - A_+(-\omega')}{\omega - \omega' - i0^+}, \tag{130}$$

where we have suppressed the tensor structure, which is identical to the one in Eq. (126).

此处我们省略了张量结构，该结构与式 (126) 中的完全一致。

The dispersion relation implies that the imaginary part of $G^R(\omega)$, responsible for dissipation, is controlled by the Wightman function $A_+(\omega)$ which we obtained by matching the microscopic theory. On the other hand, the real part of the dispersion relation

色散关系表明，负责耗散的 $G^R(\omega)$ 虚部由我们通过微观理论匹配得到的威曼函数 $A_+(\omega)$ 控制。另一方面，色散关系的实部

$$\mathrm{Re}\, G^R(\omega) = \Pr\int_0^{\infty} \frac{d\omega'\omega'}{\pi} \frac{A_+(\omega') - A_+(-\omega')}{\omega^2 - \omega'^2}, \tag{131}$$

involves arbitrarily large frequency scales. Evaluation of the RHS becomes impossible while remaining in the regime of validity of the EFT. In any case, in the EFT, the real part of the retarded response also receives a contribution from local counterterms on the black hole's worldline, e.g., from those in Eq. (16), so matching the physical response of the black hole requires knowledge of other observables beyond the graybody factor $\sigma_{abs}(\omega)$. One would also need an analytic expression for the elastic graviton scattering amplitude, in the $\omega \to 0$ limit.

包含任意大的频率标度。在保持有效场论有效区间的前提下，无法计算右侧。无论如何，在有效场论中，推迟响应的实部也会得到黑洞世界线上局部抵消项的贡献，例如式 (16) 中的抵消项，因此匹配黑洞的物理响应需要知道灰体因子 $\sigma_{abs}(\omega)$ 之外的其他可观测量，还需要得到 $\omega \to 0$ 极限下弹性引力子散射振幅的解析表达式。

We can get around this by noting that, in light of Eq. (26), we can think of the static Love number as the $\omega \to 0$ limit of the AC response function $G^R(\omega)$. Equation (26) is actually a special case of a more general linear response relation [21, 52, 59] , between the induced moment $\langle I_{ab}\rangle$ on the surface of the black hole (which, e.g., determines the long-distance quadrupolar gravitational field as seen by asymptotic observers) and the input $E_{ab}(x(\tau))$, a slowly varying background field in which the black hole propagates (Classical general relativity approaches to the motion of black holes in curved spacetime backgrounds are reviewed in [94].)

我们可以通过以下方式绕过这一问题: 根据式 (26), 我们可以将静态洛夫数视为交流响应函数 $G^R(\omega)$ 的 $\omega \to 0$ 极限。式 (26) 实际上是更一般的线性响应关系 [21, 52, 59] 的特例，该关系关联了黑洞表面的感应矩 $\langle I_{ab} \rangle$ (例如，它决定了渐近观测者观测到的长程四极引力场) 和输入项 $E_{ab}(x(\tau))$，即黑洞运动所在的缓变背景场 (弯曲时空背景下黑洞运动的经典广义相对论研究综述见文献 [94])。

$$\langle I_{ab}(\omega) \rangle = \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle I_{ab}(\tau) \rangle = \frac{2}{3}\frac{r_s^5}{G_N} k^E_{\ell=2} E_{ab}(\omega) + G_R^{ab,cd}(\omega) E_{cd}(\omega)$$

$$\equiv \frac{2}{3}\frac{r_s^5}{G_N} \kappa^E_{\ell=2}(\omega) E_{ab}(\omega), \tag{132}$$

where $E_{ab}(\omega) = \int d\tau e^{i\omega\tau} E_{ab}(x(\tau))$ and we have defined, generalizing Eq. (26), the AC Love number of the $d = 4$ Schwarzschild black hole, $\kappa^E_{\ell=2}(\omega)$ on the second line of this equation. The static Love number is then the $\omega \to 0$ limit of this quantity:

其中 $E_{ab}(\omega) = \int d\tau e^{i\omega\tau} E_{ab}(x(\tau))$，我们推广式 (26)，在本式第二行定义了 $d = 4$ 史瓦西黑洞的交流洛夫数 $\kappa^E_{\ell=2}(\omega)$。静态洛夫数就是该量的 $\omega \to 0$ 极限:

$$\kappa^E_{\ell=2}(0) = k^E_{\ell=2} + 3G_N r_s^{-5} \int_0^{\infty} \frac{d\omega}{2\pi} \frac{A_+(\omega)}{\omega}. \tag{133}$$

Calculations in full general relativity [10,27], and in the EFT [78], indicate that for Schwarzschild black holes, the quadrupolar static response vanishes, i.e.,

完整广义相对论 [10,27] 和有效场论 [78] 的计算均表明，对于史瓦西黑洞，四极静态响应为零，即

$$\kappa^E_{\ell=2}(0) = 0, \tag{134}$$

at least in $d = 4$ spacetime dimensions. This pattern seems to extend to the static Love response at $\ell > 2$ [89].

至少在 $d = 4$ 维时空下成立。该规律似乎也适用于 $\ell > 2$ 处的静态勒夫响应 [89]。

If $\kappa^E_{\ell=2}(0) = 0$, we can interpret Eq. (133) as a sum rule [101] enforcing a cancellation between the bare Love number $k^E_{\ell=2}$ and the radiative corrections from the degrees of freedom localized on the worldline. Given that the latter quantity receives contributions from UV modes of arbitrarily large frequency, this suggests that the vanishing of the static response requires a fine-tuning of parameters that is unnatural (in the t'Hooft [110] sense) from the point of view of the EFT [99, 101]. The question of whether this apparent tuning of parameters has a resolution in terms of some hidden symmetries of the EFT degrees of freedom has been a subject of recent active study; see [22, 23, 71, 72] .

若 $\kappa^E_{\ell=2}(0) = 0$，我们可以将式 (133) 解读为一个求和规则 [101]，它要求裸洛夫数 $k^E_{\ell=2}$ 和世界线局域自由度的辐射修正之间相互抵消。鉴于后者会得到任意大频率紫外模式的贡献，这说明静态响应为零要求对参数进行精细调谐，从有效场论的角度来看，这 (在特霍夫特 [110] 的意义上) 是不自然的 [99,101]。这种参数调谐是否可以通过有效场论自由度的某种隐藏对称性解决，这一问题近来是活跃研究的主题；参见 [22, 23, 71, 72] 。

Given that the static response is zero, we now have enough information to fix the low frequency AC Love number:

已知静态响应为零，我们现在已有足够信息确定低频交流洛夫数:

$$\kappa^{E}_{\ell=2}(\omega) \approx -\frac{i\pi}{15} r_s\, |\omega| + \cdots . \tag{135}$$

Inserting this into the box diagram, the instantaneous force on each black hole is local in time, but not derivable from the variation of a local action

将其代入箱图后，每个黑洞受到的瞬时力是时间局域的，但无法从局域作用量的变分导出

$$\mathbf{F}_1 = -\mathbf{F}_2 = -\frac{32}{5}\frac{G_N^7 (m_1 m_2)^2 \left(m_1^4 + m_2^4\right)}{|\mathbf{x}_1 - \mathbf{x}_2|}\left[\mathbf{v}_{12} + \frac{2\left(\mathbf{v}_{12}\cdot\mathbf{x}_{12}\right)\mathbf{x}_{12}}{|\mathbf{x}_{12}|^2}\right], \tag{136}$$

where $\mathbf{x}_{12}$ and $\mathbf{v}_{12}$ are the relative displacement and velocity of the binary. This is a 6.5PN correction to the equations of motion, analogous to the Burke-Thorne radiation damping force from the previous section. It gives a 4PN correction to the mechanical energy loss of the binary $\dot{E} = \sum_A \mathbf{v}_A \cdot \mathbf{F}_A$.

其中 $\mathbf{x}_{12}$ 和 $\mathbf{v}_{12}$ 分别是双星的相对位移和相对速度。这是运动方程的 6.5PN 阶修正，类似于上一节的伯克-索恩辐射阻尼力。它给出了双星机械能损失的 4PN 阶修正 $\dot{E} = \sum_A \mathbf{v}_A \cdot \mathbf{F}_A$ 。

In the case of spinning objects, the correlator is no longer determined by a single form factor $A_+^{E,B}(\omega)$ as more tensor structures, involving the spin vector of the object, can appear. For non-zero spin, the EFT has been extended to slowly spinning black holes [96], to more general spinning sources in Ref. [37], and generalized to rapidly spinning (close to extremal) Kerr black holes [60]. As in the non-spinning case, the full causal response is determined by matching to the black hole absorption rates in [87, 107] as well as results of [26, 81, 82] , to fix the local counterterms (the conservative part of the tidal response) on the worldline Lagrangian to zero.

对于旋转天体，关联函数不再由单一形状因子 $A_+^{E,B}(\omega)$ 决定，因为会出现更多包含天体自旋矢量的张量结构。对于非零自旋，有效场论已经被推广到慢旋转黑洞 [96]，文献 [37] 将其推广到更一般的旋转源，又进一步推广到快旋转 (近极端) 克尔黑洞 [60]。与非旋转情况一样，完整因果响应通过匹配 [87, 107] 中的黑洞吸收率以及 [26, 81, 82] 的结果来确定，从而将世界线拉格朗日量中的局部抵消项 (潮汐响应的保守部分) 固定为零。

For instance, the energy loss in the spinning case, for arbitrary spins

例如，任意自旋下旋转情形的能量损失

$$\left.\frac{dE}{dt}\right|_h = \frac{8}{5}\frac{G_N^5 m_1^2 m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}(m_1 + m_2)\left[1 + 3\chi_1^2 - \frac{15}{4}\chi_1^2\left(\frac{\mathbf{s}_1\cdot(\mathbf{x}_1 - \mathbf{x}_2)}{|\mathbf{x}_1 - \mathbf{x}_2|}\right)^2\right]\mathbf{S}_1\cdot\mathbf{L} + (1 \leftrightarrow 2), \tag{137}$$

at leading PN order [60] ($\mathbf{s}_A = \mathbf{S}_A / |\mathbf{S}_A|$). For nearly maximally rotating black holes, with $\chi = |\mathbf{S}| / G_N M \lesssim 1$, Eq. (137) gets enhanced, by the superradiant effect [107, 118], to 2.5PN order compared to 4PN in the Schwarzschild case, $\chi = 0$. Notice that the energy flux can have either sign depending on the relative orientation between the black hole spin and the orbital angular momentum $\mathbf{L}$. In particular, it is possible to extract rotational energy from the black holes as in the Penrose process [91]. Equation (137) generalizes to arbitrary orbits and spin orientations earlier results obtained by classical techniques in [5, 24, 25] .

处于 PN 领头阶 [60] ($\mathbf{s}_A = \mathbf{S}_A / |\mathbf{S}_A|$)。对于近最大旋转黑洞，满足 $\chi = |\mathbf{S}| / G_N M \lesssim 1$ ，式 (137) 因超辐射效应 [107, 118] 得到增强，对应的是 2.5PN 阶，而施瓦西情形为 4PN 阶，$\chi = 0$。注意能量通量的符号可正可负，取决于黑洞自旋与轨道角动量的相对方向 $\mathbf{L}$。具体来说，我们可以像彭罗塞过程 [91] 一样从黑洞提取转动能。式 (137) 推广了早前通过经典方法在 [5, 24, 25] 中得到的、适用于任意轨道和自旋取向的结果。

Quantum Effects

量子效应

Because the TOU formalism is explicitly quantum mechanical, it is capable of describing processes involving quantum black holes [67, 68] interacting with other particles or fields. In particular, it can be used [57,58] to take into account the effects of Hawking radiation on scattering observables.

由于 TOU 形式主义是显式量子力学框架，它能够描述涉及量子黑洞 [67, 68] 与其他粒子或场相互作用的过程。具体而言，它已被用于 [57,58] 计入霍金辐射对散射观测量的影响。

The EFT description assumes that black holes behave according to the standard quantum mechanical rules, with unitary time evolution and a complete Hilbert space of microstates. It is valid in the window of momentum transfers q defined by

EFT 描述假设黑洞遵循标准量子力学规则演化，具有幺正时间演化和完备的微观态希尔伯特空间。该描述在由以下条件定义的动量转移窗口 q 内成立

$$t_{\text{Page}}^{-1} \ll q \ll 1/G_N M_{BH} \ll m_{Pl}, \tag{138}$$

where, e.g., in the Schwarzschild case, the black hole evaporation scale is of order the Page time [87], $t_{\text{Page}} \sim G_N^2 M_{BH}^3$. The upper bound ensures that the worldline description of the black hole is reliable, while the lower bound allows us to treat black holes as approximately long-lived asymptotic states in the S -matrix. Since $G_N M_{BH} \gg 1/m_{PL}$, the black holes can be regarded as being semiclassical. Because the time scales are short compared to t_{Page} , the processes we will consider are not sensitive to the apparent loss of unitarity in black hole evaporation [68].

例如在史瓦西情形中, 黑洞蒸发尺度为佩奇时间量级 [87], $t_{\text{Page}} \sim G_N^2 M_{BH}^3$。上界保证黑洞的世界线描述是可靠的，而下界允许我们将黑洞视为 S 矩阵中近似长寿命的渐近态。由于 $G_N M_{BH} \gg 1/m_{PL}$ ，黑洞可以被视为半经典对象。由于我们考虑过程的时间尺度远短于 t_{Page} ，过程不会受到黑洞蒸发中明显的幺正性损失的影响 [68]。

To construct the EFT, we have to determine how the structure of multipole operator correlation functions is modified by the presence of a Hawking thermal spectrum of radiation emission from the black hole. As in the classical case, we match to the simplest possible observables that depend, on the EFT side, on the multipole correlators. It is convenient in particular to calculate the on-shell transition probabilities $p(m \to n)$ for a Kerr black hole to emit n gravitons out to future $\mathcal{I}^+$, given that m particles of the same energy ω and angular momentum quantum number ℓ are incident on the black hole in the far past from $\mathcal{I}^-$. Explicit results (The results of Refs. [8, 88] are for free scalar fields propagating in the black hole background, but they generalize naturally to higher spin $s > 0$ fields by simply replacing the scalar transmission coefficients by their higher spin version found in Refs. [87, 107].) are available for this observable, in the limit of free quantum field theory in the black hole background [8, 88]

为构造 EFT，我们需要确定黑洞发射霍金热辐射谱如何改变多极算符关联函数的结构。与经典情形一样，我们将其匹配到依赖于 EFT 侧多极关联函数的最简单观测量。具体而言，计算克尔黑洞的如下在壳跃迁概率 $p(m \to n)$ 十分方便: 在遥远过去从 $\mathcal{I}^-$ 入射带有相同能量 ω 和角动量量子数 ℓ 的 m 粒子的条件下，克尔黑洞向未来 $\mathcal{I}^+$ 出射 n 引力子的在壳跃迁概率。该观测量已有明确结果 (文献 [8, 88] 的结果针对在黑洞背景中传播的自由标量场，但可以自然推广到高自旋 $s > 0$ 场，只需将标量透射系数替换为文献 [87, 107] 中给出的高自旋透射系数即可)，结果适用于黑洞背景下自由量子场论极限 [8, 88]

$$p_\ell(m \to n) = \frac{(1-x)\, x^n \left(1 - |R_\ell|^2\right)^{n+m}}{\left(1 - x|R_\ell|^2\right)^{n+m+1}} \tag{139}$$
$$\times \sum_{k=0}^{\min(n,m)} \frac{(n+m-k)!}{k!\,(n-k)!\,(m-k)!} \left[\frac{\left(x^{-1}|R_\ell|^2 - 1\right)\left(1 - x|R_\ell|^2\right)}{\left(1 - |R_\ell|^2\right)^2}\right]^k ,$$

where $x = \exp[-\beta_H \omega]$, $|R_\ell(\omega)|^2$ is the Boltzmann factor for the Kerr black hole and $|R_\ell(\omega)|^2$ are the partial wave reflection coefficients obtained in [87, 107] .

其中 $x = \exp[-\beta_H \omega]$, $|R_\ell(\omega)|^2$ 是克尔黑洞的玻尔兹曼因子，$|R_\ell(\omega)|^2$ 是在 [87, 107] 中得到的分波反射系数。

In ZSOU, one finds that to leading order in $\omega/m_{Pl} \ll 1$, the $m \to n$ - particle probabilities are controlled by $n+m$ -point Wightman correlation functions of the multipole operators in Eq. (57). Given the structure of the full theory result [8, 88] , these higher points correlators factorize into suitable products of two-point functions, up to non-Gaussianities suppressed by $\omega^2/m_{Pl}^2 \ll 1$. Somewhat surprisingly, one finds that at $r_s\omega \ll 1$ the effects of Hawking radiation are not Planck suppressed at the level of the Wightman functions. Instead, they become enhanced in the limit $\hbar\omega/T_H = 4\pi r_s \omega \ll 1$ where the EFT is valid, a consequence of the high temperature behavior of the Planck distribution.

在 ZSOU 中可以发现，在 $\omega/m_{Pl} \ll 1$ 的领头阶下，$m \to n$ 粒子概率由式 (57) 中多极算符的 $n+m$ 点威曼关联函数主导。在完整理论结果 [8, 88] 的结构中，这些高阶关联函数可以分解为两点函数的合适乘积，修正项是被 $\omega^2/m_{Pl}^2 \ll 1$ 压低的非高斯贡献。有些出乎意料的是，在 $r_s\omega \ll 1$ 阶，霍金辐射的效应在威曼函数层面并不被普朗克压低。相反，在 EFT 成立的 $\hbar\omega/T_H = 4\pi r_s \omega \ll 1$ 极限下，这些效应反而被增强，这是普朗克分布高温行为的结果。

Despite this enhancement at the level of the Wightman functions, in the full theory the effects of Hawking radiation cancel in the free field retarded correlators. Up to corrections suppressed by $1/m_{Pl}$, these take the same form in the Unruh state [111] that describes an evaporating black hole or the Boulware state [13] where the black hole does not emit radiation. Consequently, finite size effects associated with the quantized nature of the black hole horizon, e.g., in the observables discussed in the previous section, are suppressed by at least one power of $\omega^2/m_{PL}^2 \ll 1$. Thus, if one assumes that black holes evolve according to the usual rules of quantum mechanics, the results of [57] imply a no-go theorem on the possibility of detecting black microstate "hair" or other possible exotic signatures of quantum behavior in binary black hole mergers at LIGO/VIRGO or any other foreseeable experiment.

尽管在怀特曼函数层面存在这种增强，但在完整理论中，霍金辐射的效应在自由场推迟关联函数中相互抵消。在 $1/m_{Pl}$ 压低修正范围内，无论是描述蒸发黑洞的安鲁态 [111]，还是黑洞不辐射的博尔沃态 [13]，这些关联函数都保持相同形式。因此，与黑洞视界量子化性质相关的有限尺寸效应 (例如上一节讨论的可观测效应中的这类效应) 至少会被 $\omega^2/m_{PL}^2 \ll 1$ 的一次幂压低。由此可见，如果假设黑洞遵循常规量子力学规则演化，文献 [57] 的结论就意味着一条禁戒定理: 在 LIGO/VIRGO 或任何其他可预见的实验中，不可能探测到黑洞微态“毛”，或是双黑洞合并中量子行为的其他可能奇特信号。

On the other hand, the emission of Hawking radiation does modify observables that depend on the Wightman functions directly. While not of phenomenological important, an example [58] of formal interest is the inelastic scattering of elementary particles incident on a semiclassical black hole, mediated by the exchange of virtual (off-shell) Hawking gravitons.

另一方面，霍金辐射的发射确实会直接改变依赖于怀特曼函数的可观测量。尽管不具备现象学研究价值，一个具有形式理论意义的例子 [58] 是: 入射到半经典黑洞的基本粒子发生非弹性散射，该过程由虚 (离壳) 霍金引力子交换介导。

For illustration, consider a scalar particle ϕ with mass in the range $k_B T_H \ll m_\phi \ll M_{BH}$. In this window, direct s -channel production of ϕ -particle Hawking pairs is exponentially (Boltzmann) suppressed, so that the scattering process

举例来说，考虑一个质量在范围 $k_B T_H \ll m_\phi \ll M_{BH}$ 内的标量粒子 ϕ 。在该范围内，直接通过 s 道产生 ϕ 粒子霍金对会受到玻尔兹曼指数抑制，因此散射过程

$$\phi(p) + \mathrm{BH} \to \phi(p') + \mathrm{BH}' \tag{140}$$

proceeds instead through off-shell graviton exchange. One finds the result [58]

反而通过离壳引力子交换进行。文献 [58] 得到结果

$$\frac{d^3\sigma}{dq^2 d(q\cdot v)} \approx \frac{7G_N r_s^5}{270\pi\left[(v\cdot p)^2 - m_\phi^2\right]}\left[(v\cdot p)^4 - m_\phi^2 (v\cdot p)^2\left(1 - \frac{12}{7}\frac{(v\cdot q)^2}{q^2}\right)\right.$$
$$\left. + \frac{1}{7}m^4\left(1 - 3\frac{(v\cdot q)^2}{q^2} + 6\frac{(v\cdot q)^4}{q^4}\right)\right], \tag{141}$$

for the differential cross section in the black hole rest frame $v^\mu = (1, \mathbf{0})$, as a function of the momentum transfer $q^\mu = p^\mu - p'^\mu$. The point of this result is that, in generic regions of phase space, the integrated cross section scales as $\sim q^2/m_{Pl}^2$ relative to the leading order classical gravitational scattering between point masses M, m_ϕ . It is parametrically of the same size as the sort of $\mathcal{O}(\hbar)$ corrections from graviton vacuum polarization loops that appear in scattering processes involving elementary particles, e.g., [11,33,34]. Equation (141) can be therefore be regarded as a specific realization of a qualitatively new phenomenon in low energy quantum gravity, associated with the exchange of virtual Hawking gravitons and tractable (calculable) by the methods of effective field theory applied to gravity.

用于黑洞静止系中的微分截面，是动量转移 $q^\mu = p^\mu - p'^\mu$ 的函数 $v^\mu = (1, \mathbf{0})$ 。该结果的核心意义在于，相空间的一般区域中，积分截面相对于点质量之间领头阶经典引力散射的标度关系为 $\sim q^2/m_{Pl}^2$ M, m_ϕ 。从参数上看，它的大小与基本粒子散射过程中出现的引力子真空极化圈带来的 $\mathcal{O}(\hbar)$ 修正量级相同，例如见文献 [11,33,34]。因此，方程 (141) 可以看作低能量子引力中一种全新定性现象的具体实现；这种现象与虚拟霍金引力子交换相关，且可以通过应用于引力的有效场论方法处理 (计算得到)。

Cross References

交叉引用

Effective Field Theory and Applications

有效场论及应用

- Gravity, Horizons, and Open EFTs

- 引力、视界与开有效场论

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